INFORMATION FLOW IN THE RETINA - CORTEX PATH:

GAUSSIANIZATION ESTIMATES & THEORETICAL RESULTS

JESÚS MALO

VNIVERSITAT DE VALÈNCIA

SPAIN.

CNS*2020 Workshop on Methods of Information Theory in Computational Neuroscience

Organization For Computational Neurosciences

1. IMAGE QUALITY & VISION MODELS: Computational Neuroscience may make you a MOVIE STAR!

2. THE PROBLEM: Quantifying information flow

3. THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

4. EXPERIMENTS & RESULTS Efficient Coding & Image Quality

5. DISCUSSION & CONCLUSIONS

1 IMAGE QUALITY & VISION MODELS: Computational Neuroscience may make you a MOVIE STAR!

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THE IMAGE QUALITY PROBLEM

Structural Similarity (SSIM)

67th EMMY Engineering Award of the American Television Academy 2015 !

https://youtu.be/e5-LCFGdgMA

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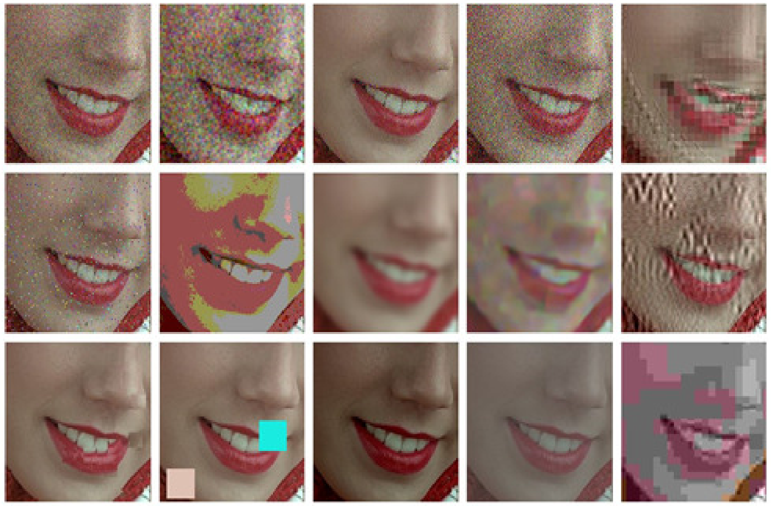
THE IMAGE QUALITY PROBLEM

x



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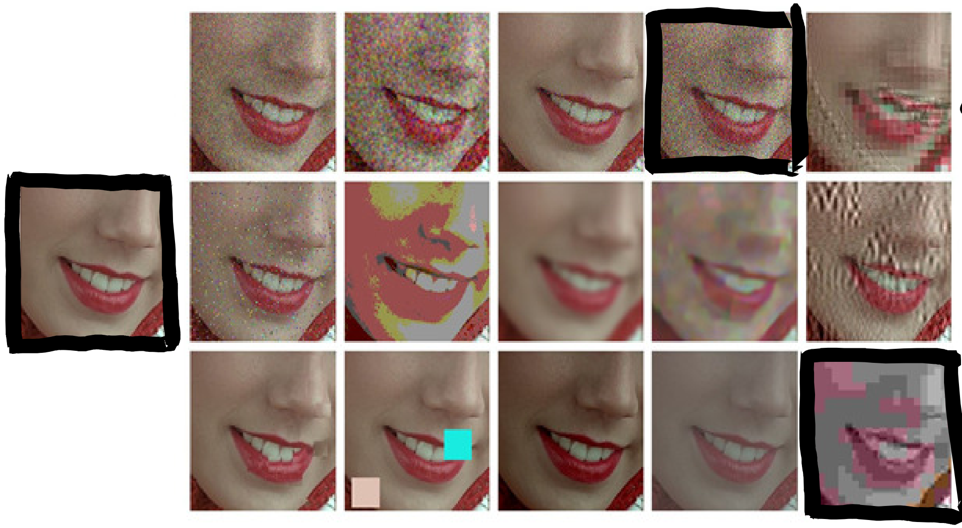

$y^{(1)} = x + \Delta x^{(2)}$

$y^{(2)} = x + \Delta x^{(1)}$

SUBJECT. RATING (M.O.S)

Structural Similarity (SSIM)

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$d(x, x + \Delta x^{(i)})$

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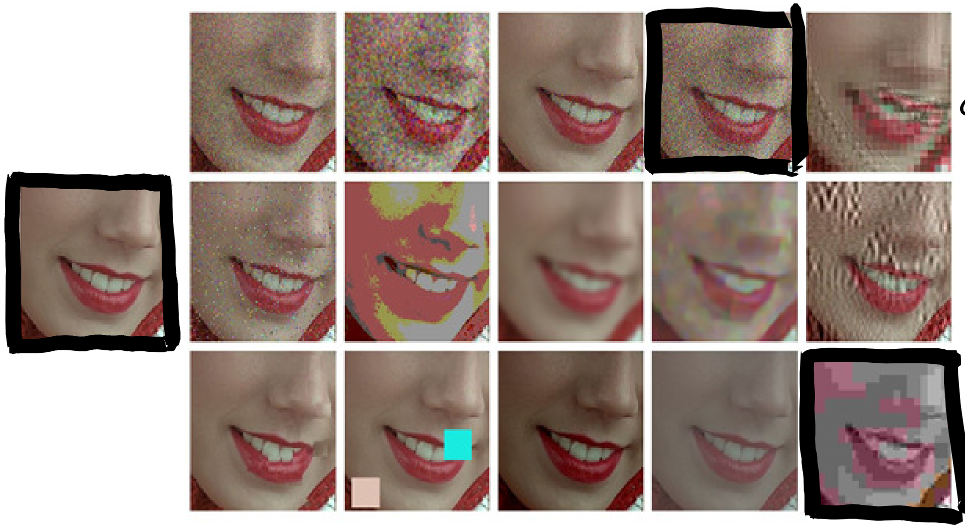
THE IMAGE QUALITY PROBLEM



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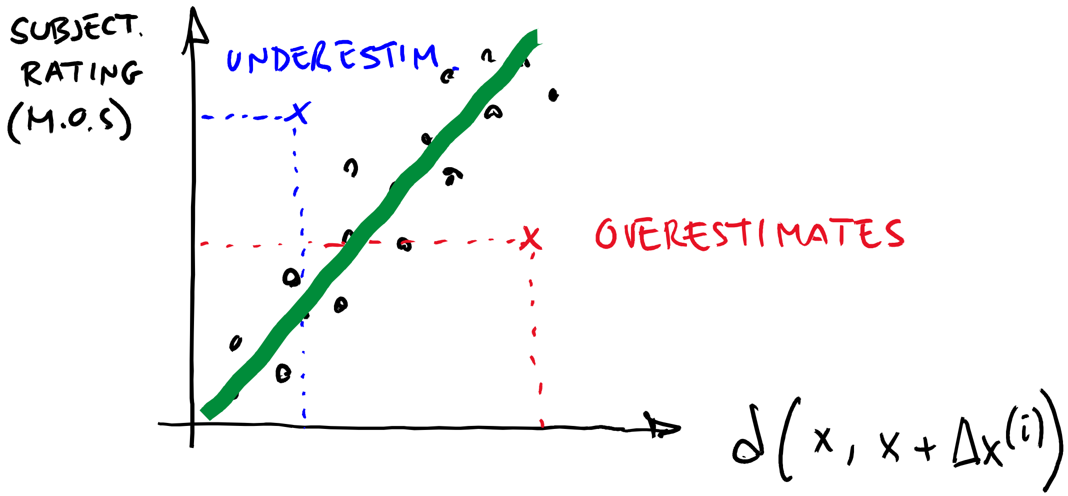

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Google Scholar

Alan Bovik

FOLLOW

Cockrell Family Regents Endowed Chair Professor, The University of Texas at Austin
Verified email at ece.utexas.edu - Homepage

Electrical Engineering Digital Television Digital Photography Social Media Image Processing

TITLE	CITED BY	YEAR
Image quality assessment: from error visibility to structural similarity Z Wang, AC Bovik, HR Sheikh, EP Simoncelli IEEE transactions on image processing 13 (4), 600-612	26470	2004
A universal image quality index Z Wang, AC Bovik IEEE Signal Processing Letters 9 (3), 81-84	5361	2002
Multiscale structural similarity for image quality assessment Z Wang, EP Simoncelli, AC Bovik Asilomar Conference on Signals, Systems & Computers 2, 1398-1402	3372	2003
Image information and visual quality HR Sheikh, AC Bovik IEEE Transactions on Image Processing 15 (2), 430-444	2959	2006

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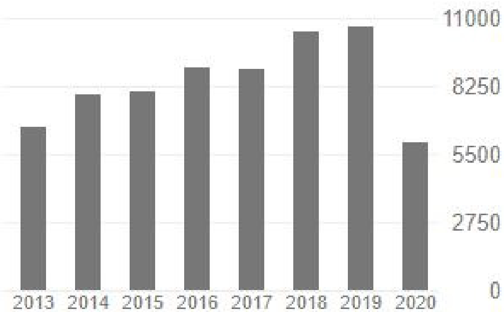
i10-index	478	304



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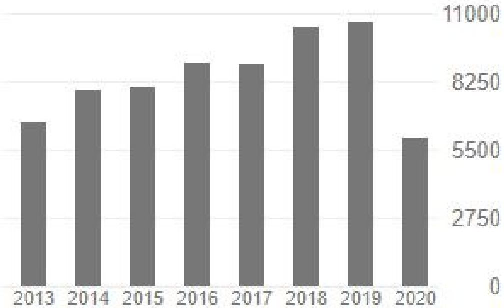

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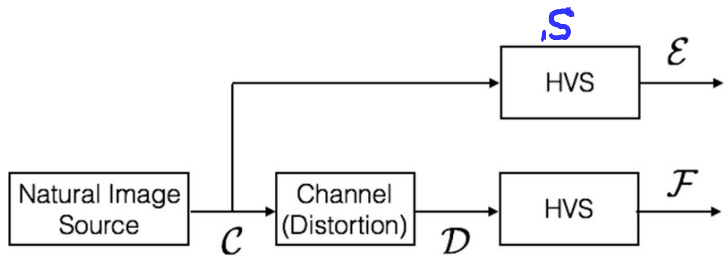

GEOMETRY ≡ DISTANCE

$$d(C,D) = |S(C) - S(D)| = |E - F|$$

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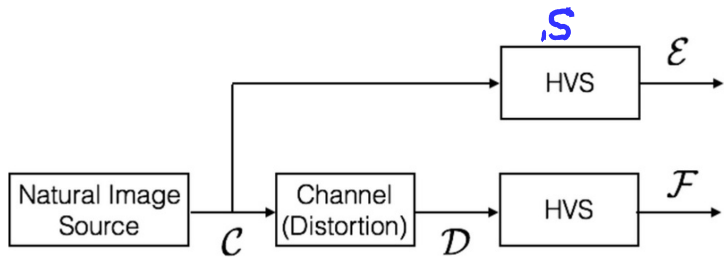

GEOMETRY ≡ DISTANCE

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A. New Philosophy

STRUCTURE

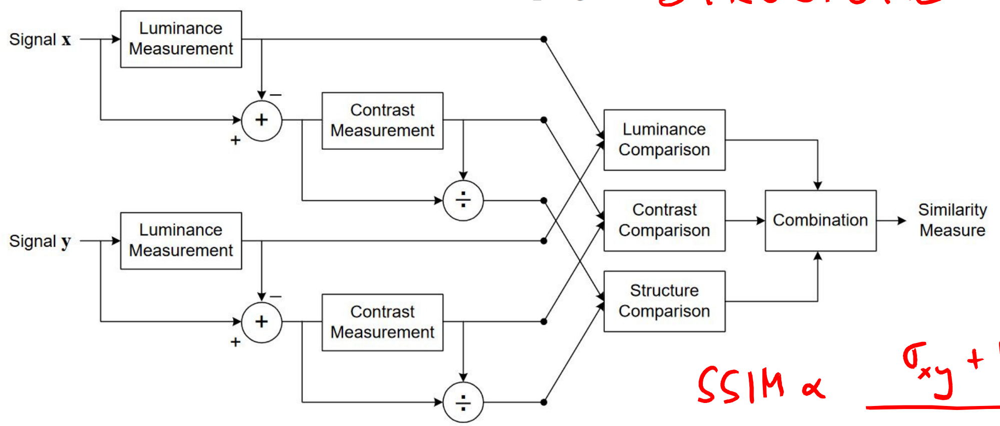

Fig. 3. Diagram of the structural similarity (SSIM) measurement system.

$$SSIM \propto \frac{\sigma_{xy} + k}{\sigma_x \sigma_y + k}$$

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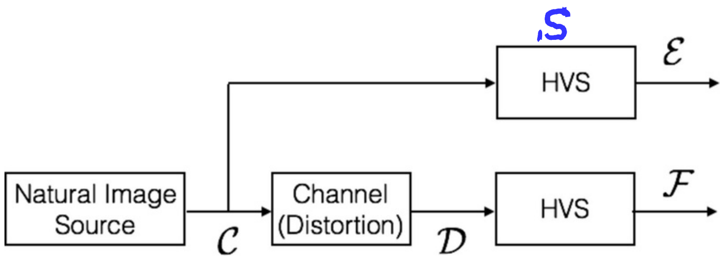

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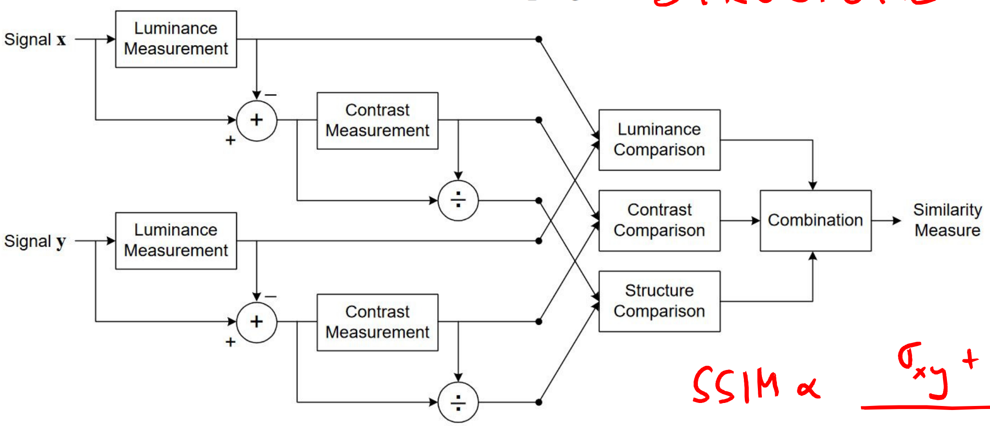
A. *New Philosophy* STRUCTURE



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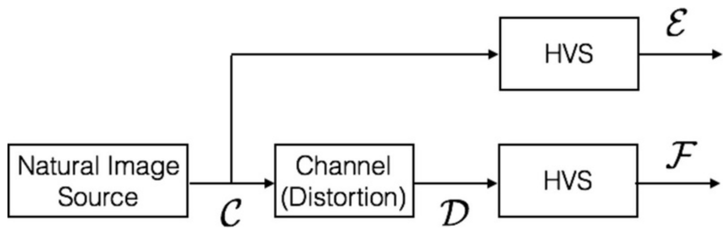
A. *New Philosophy* INFORMATION



$$VIF = \frac{I(C, S(D))}{I(C, S(C))} = \frac{I(C,F)}{I(C,E)}$$

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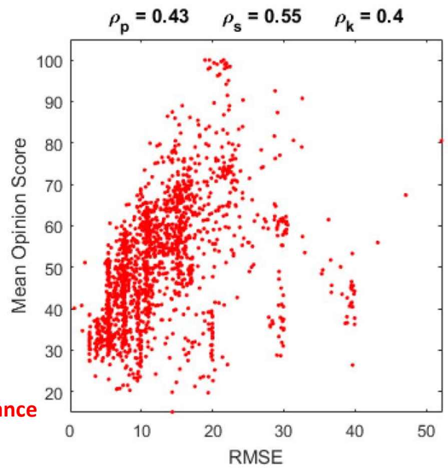
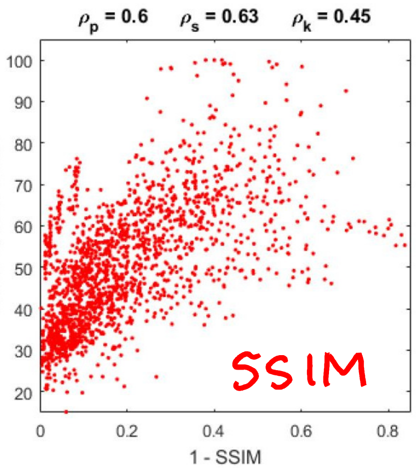


RMSE
Euclidean Distance



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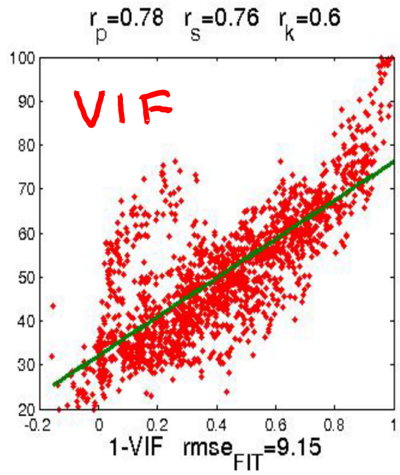
SSIM
67th EMMY Engineering Award of the American Television Academy 2015 !



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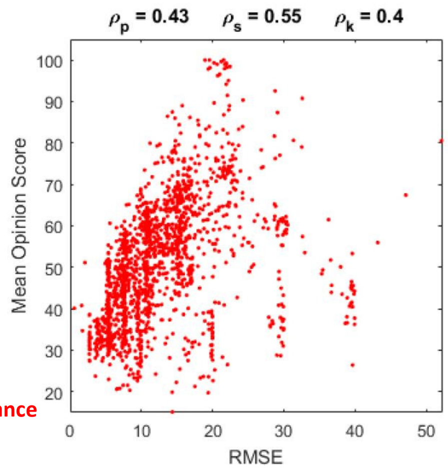

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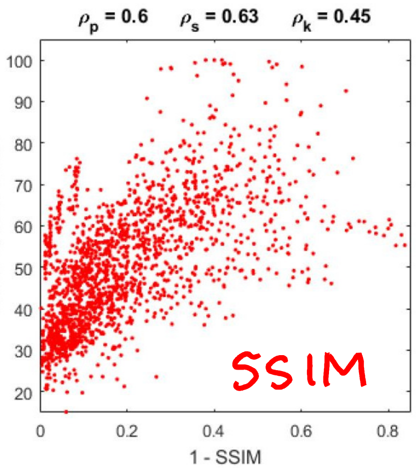
Euclidean Distance



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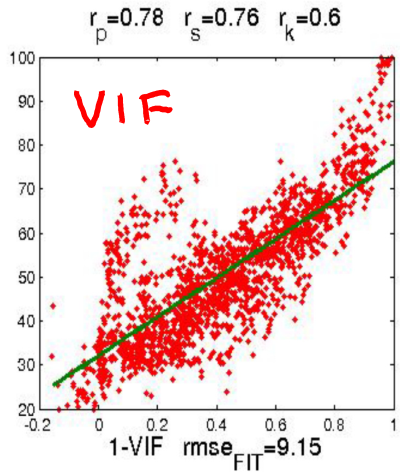
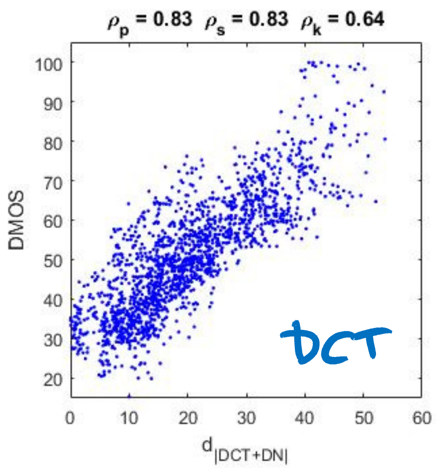
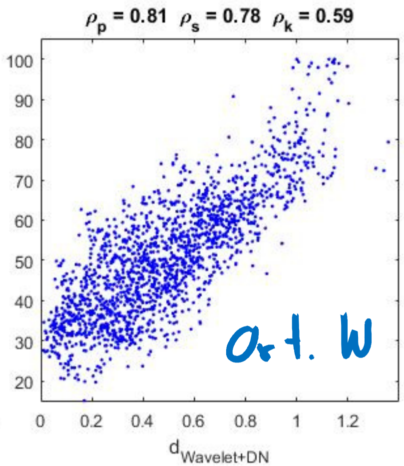




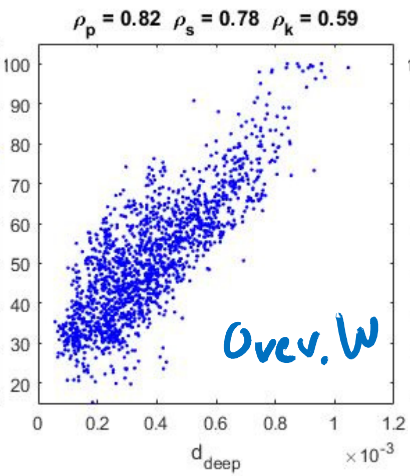
V1_model_DCT_DN_color

Im. Vis. Comp. 1997
IEEE Trans. Im. Proc. 2006 ....



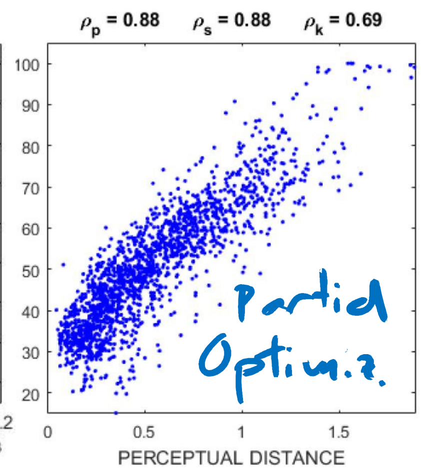
V1_model_wavelet_DN_color

JOSA A 2010
Neural Comput. 2010



BioMultiLayer_L_NL_color

Front. Neurosci. 2018 a



BioMultiLayer_L_NL_color (Optimized)

PLoS ONE 2018

DIVISIVE NORMALIZATION

Teo & Heeger 94
Malo et al. 97
Malo & Simoncelli 06
Laparra & Malo 10
Malo & Simoncelli 15
Laparra & Simoncelli 17
Malo et al. 18
Hepburn & Malo 20

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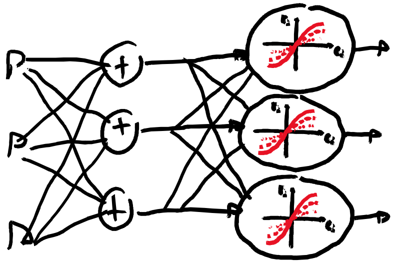
Divisive Normalization neural models

BIOL. Carandini & Heeger Nat. Rev. Neurosci 12

MATH. Martinez, Malo et al. PLOS ONE 18

$$x \xrightarrow{\mathcal{L}} e \xrightarrow{\mathcal{N}} y \qquad x \xrightarrow{S} y$$



$\mathcal{L}$

$\mathcal{N}$

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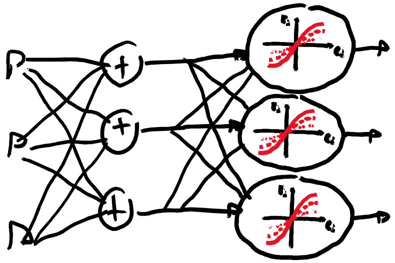
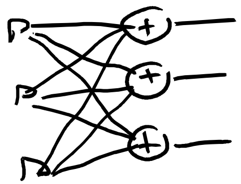
$$\begin{tikzcd} x \arrow[r, "\mathcal{L}"'] \arrow[rr, bend left, "S"] & e \arrow[r, "N"'] & y \end{tikzcd}$$



$\mathcal{L}$ $\quad e = W \cdot x$



N

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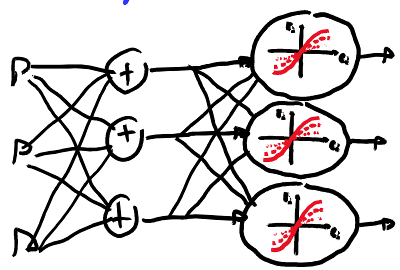
$$
\begin{tikzcd}
x \arrow[r, "\mathcal{L}"'] \arrow[rr, bend left, "S"] & e \arrow[r, "\mathcal{N}"'] & y
\end{tikzcd}
$$



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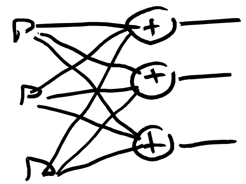


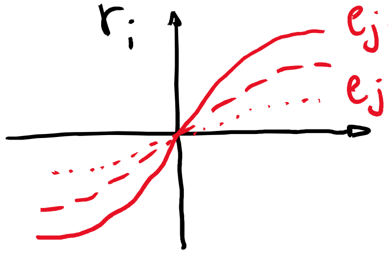
e = linear response

b = semisaturation

H = interaction Kernel

K = constant → dyn. range

$\mathcal{N}$ $\quad y = K \cdot \dfrac{e}{b + H \cdot e}$



Masking and adaptation

$$\nabla_x S \sim \left[I - D_{r(x)} H \right] \cdot D_e \cdot W \Rightarrow M = \nabla_x S^T \nabla_x S$$

NON DIAGONAL!
INPUT DEPENDENT!

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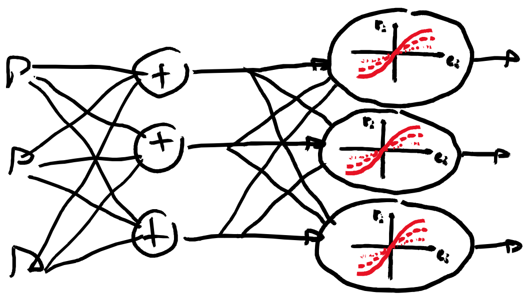
RETINA

$\mathcal{L}$ $\mathcal{N}$

Radiation LMS Von Kries



Chromatic Adaptation

V1 CORTEX

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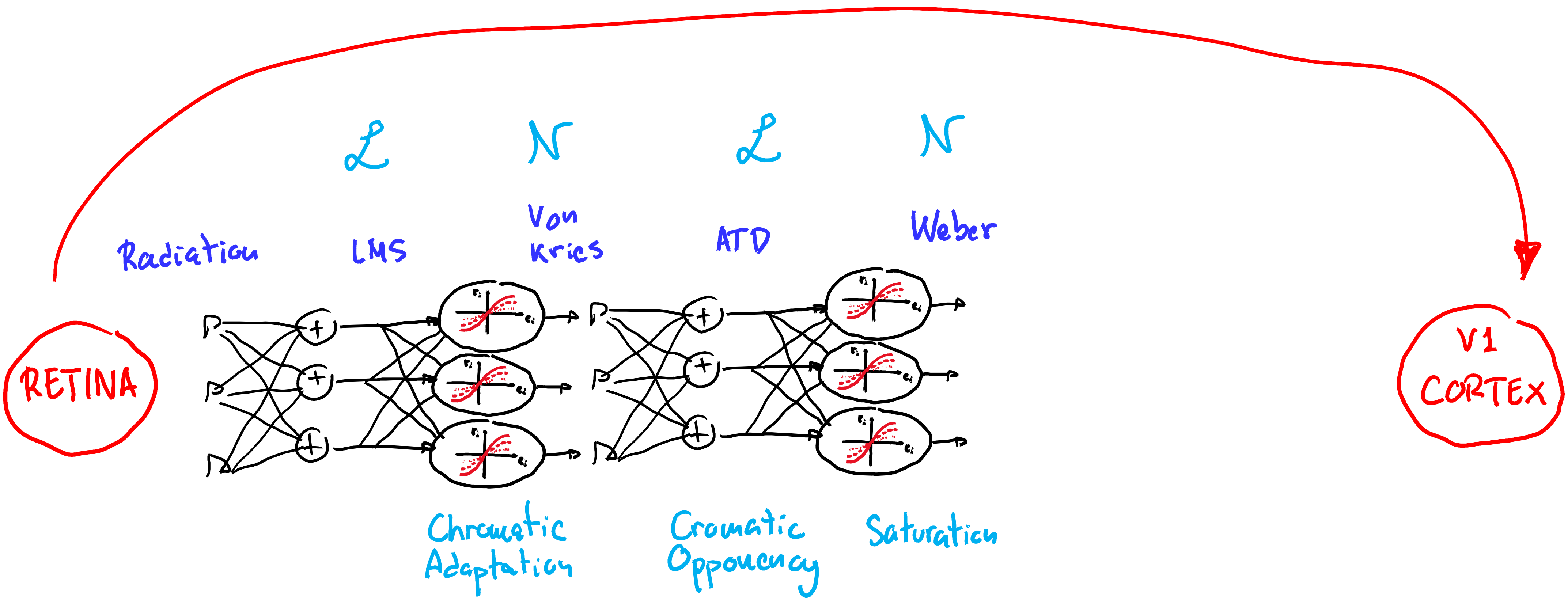

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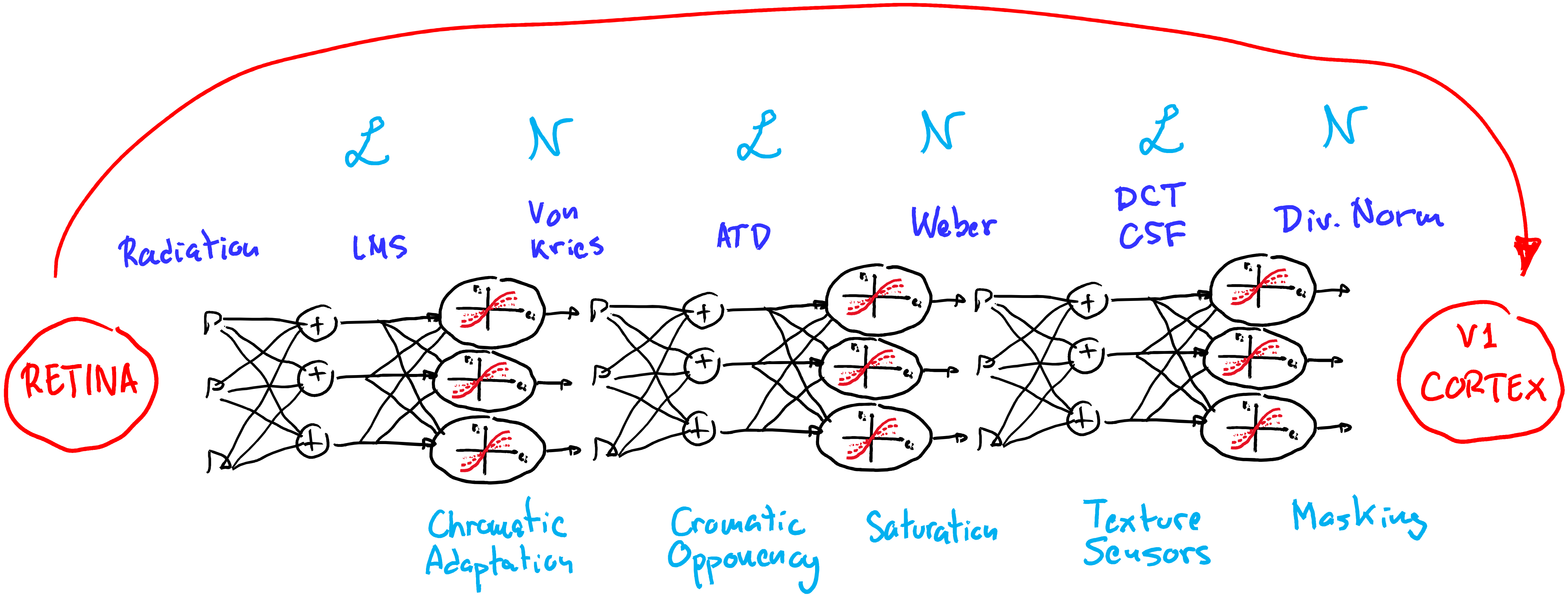

2 THE PROBLEM: Quantifying information flow

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(2) THE PROBLEM: Quantifying information flow

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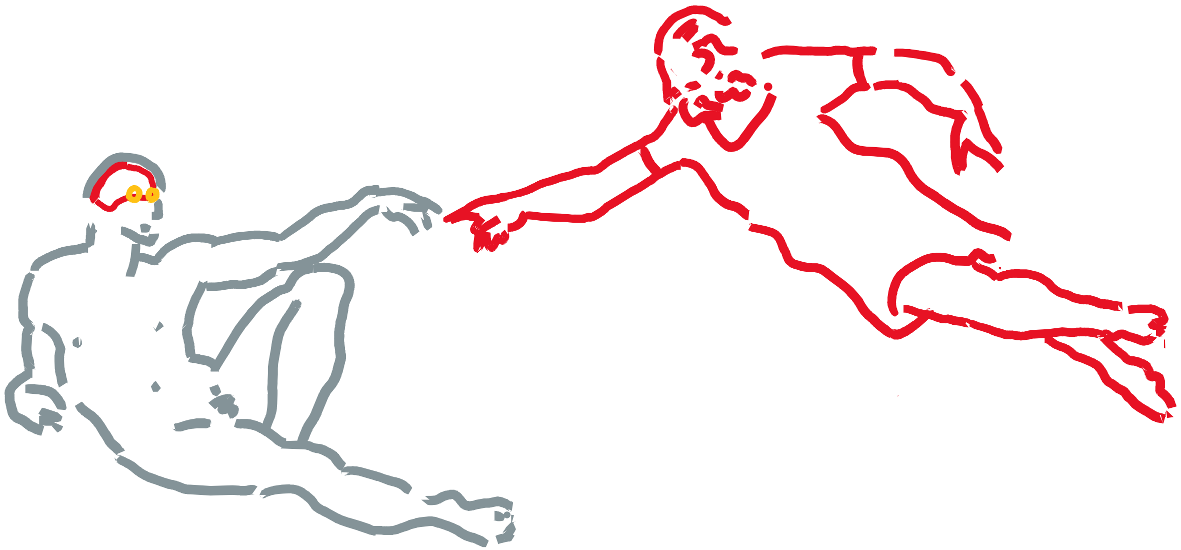

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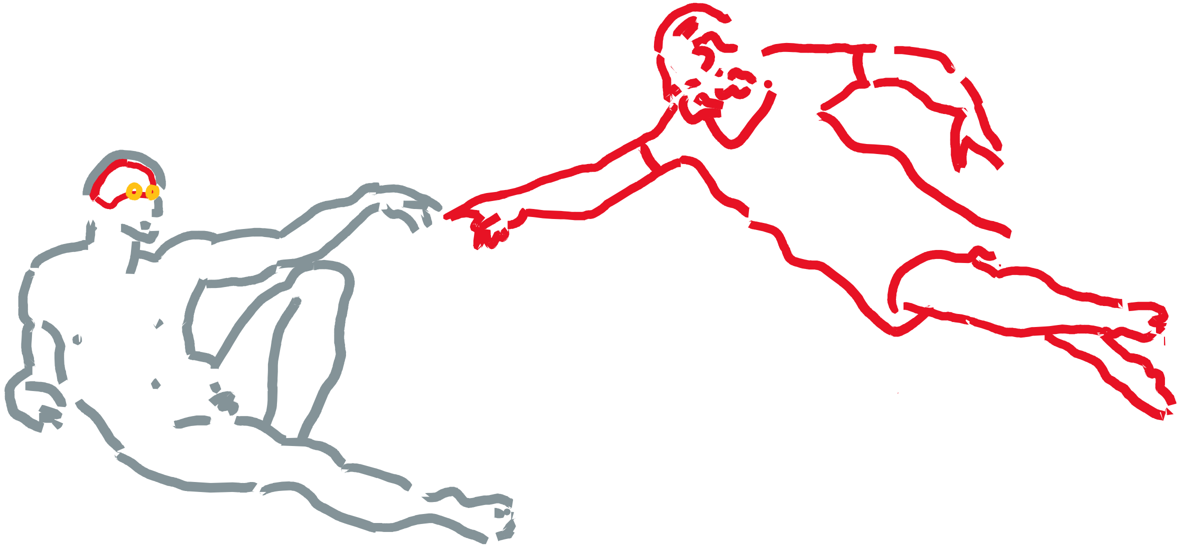

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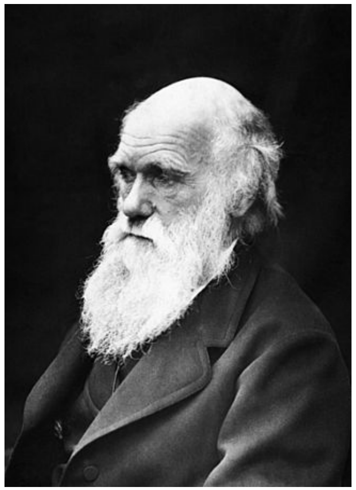

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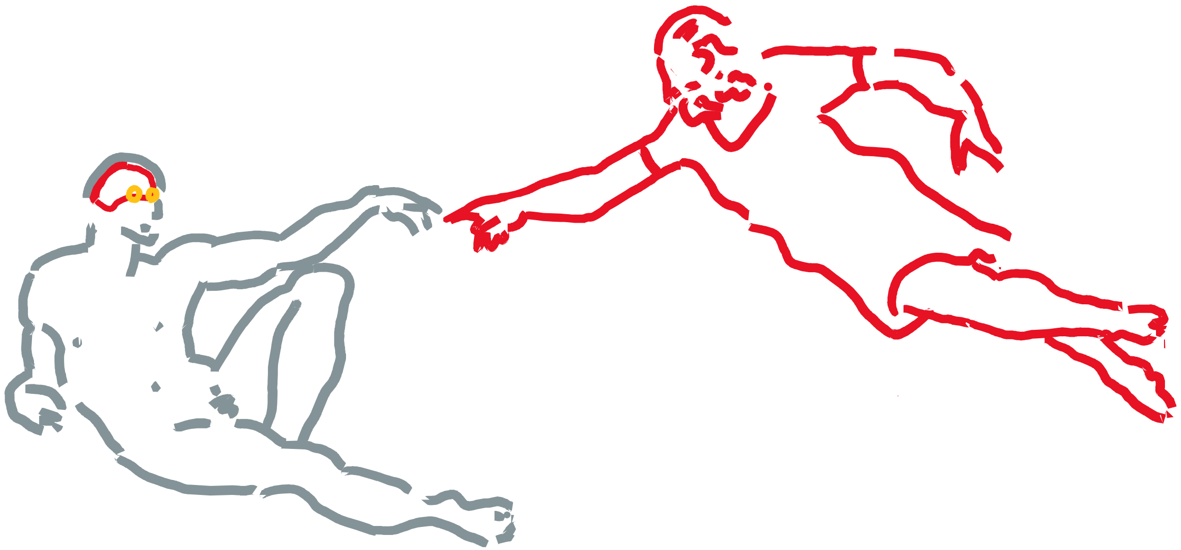

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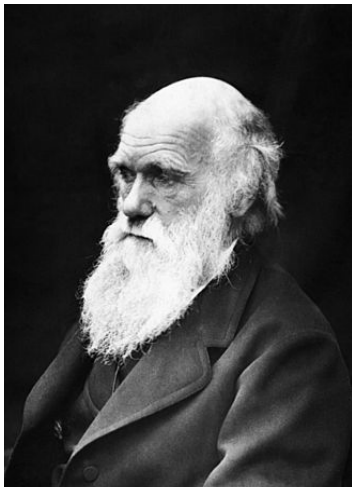

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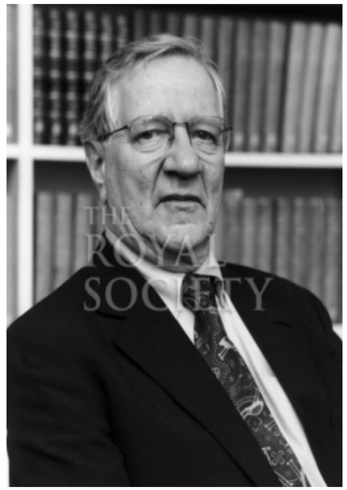

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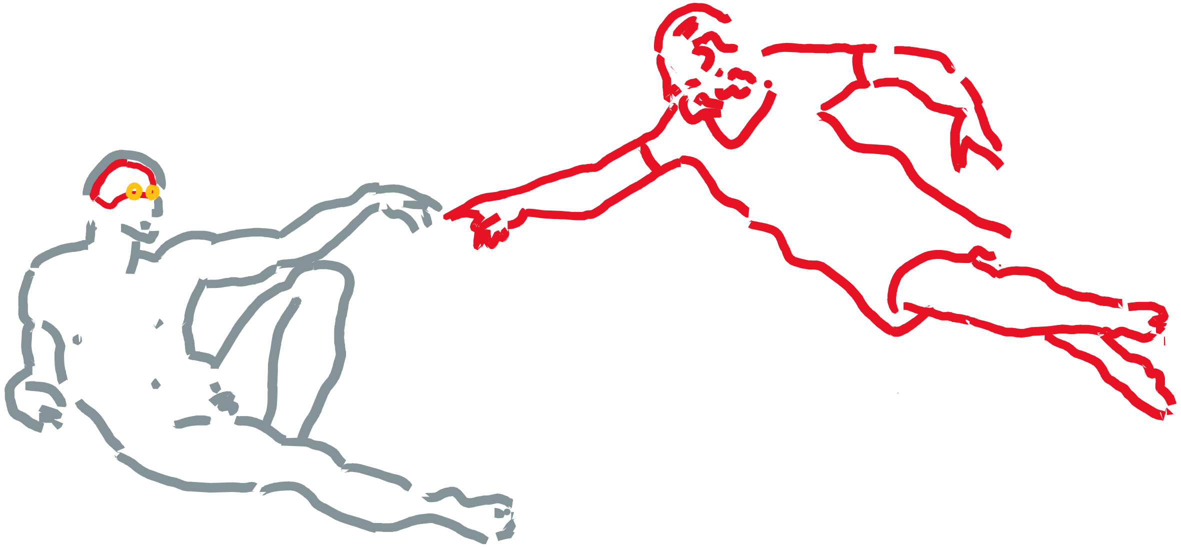

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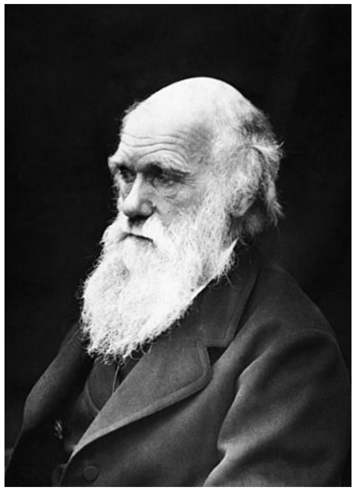
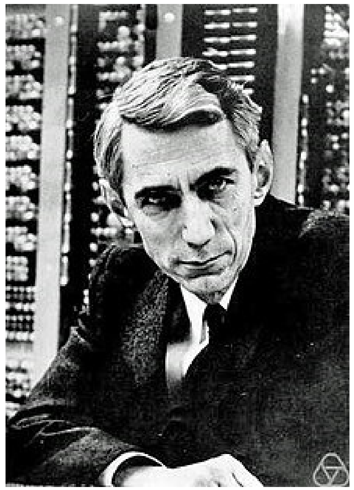


SHANNON



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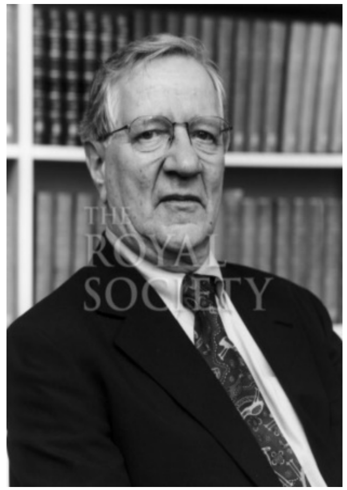

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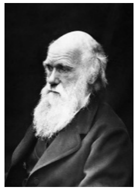
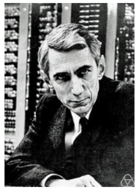
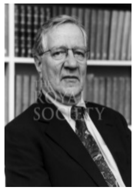
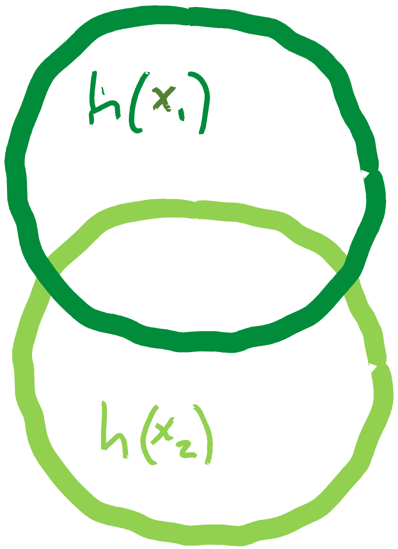
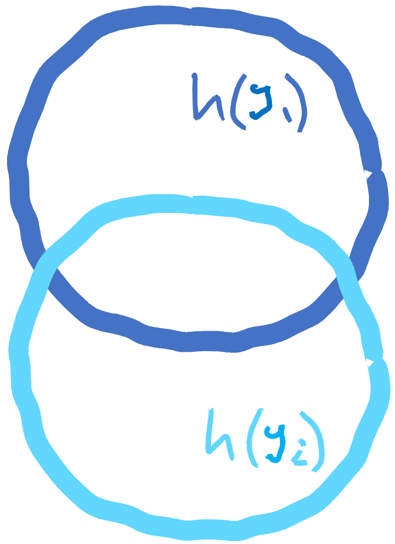






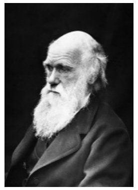
$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \xrightarrow{S_\theta} y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

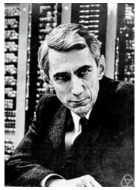


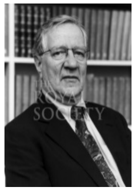


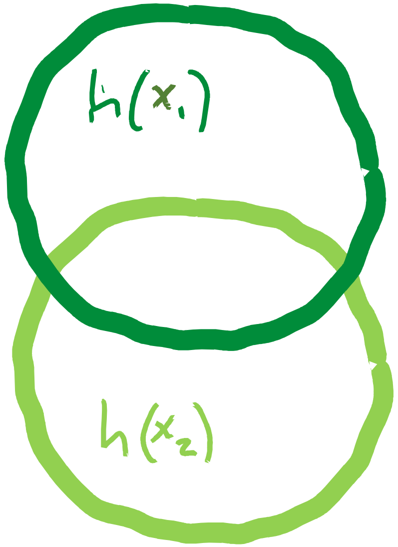
2 THE PROBLEM: Quantifying information flow

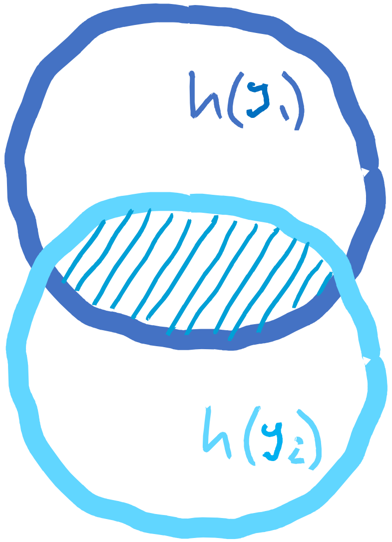
$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \xrightarrow{S_\theta} y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$











TOTAL CORRELATION ≡ Redundancy within a vector $T(y) = \sum_i h(y_i) - h(y)$

2 THE PROBLEM: Quantifying information flow

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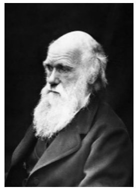

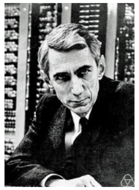

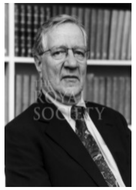

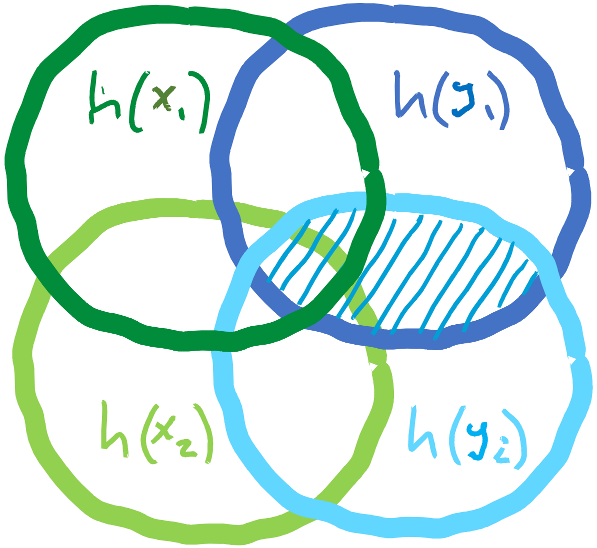

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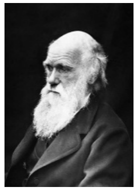
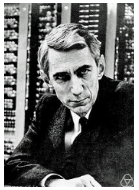
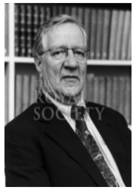
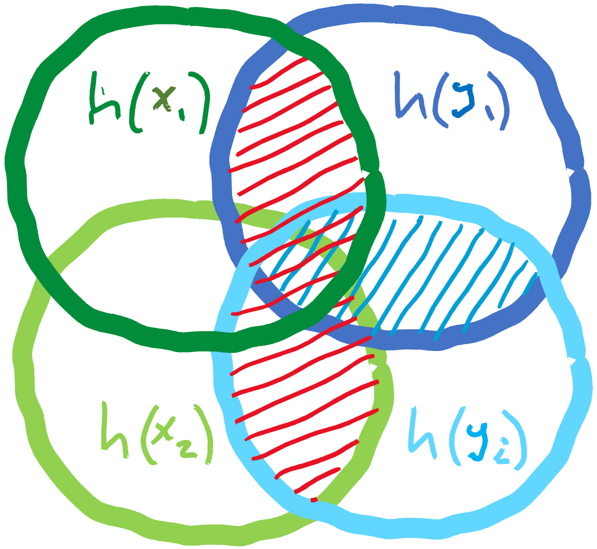
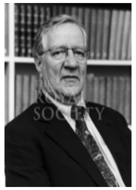






TOTAL CORRELATION ≡ Redundancy within a vector $T(y) = \sum_i h(y_i) - h(y)$

MUTUAL INFORMATION ≡ Info shared by two vectors $I(x,y) = h(x) + h(y) - h([x,y])$

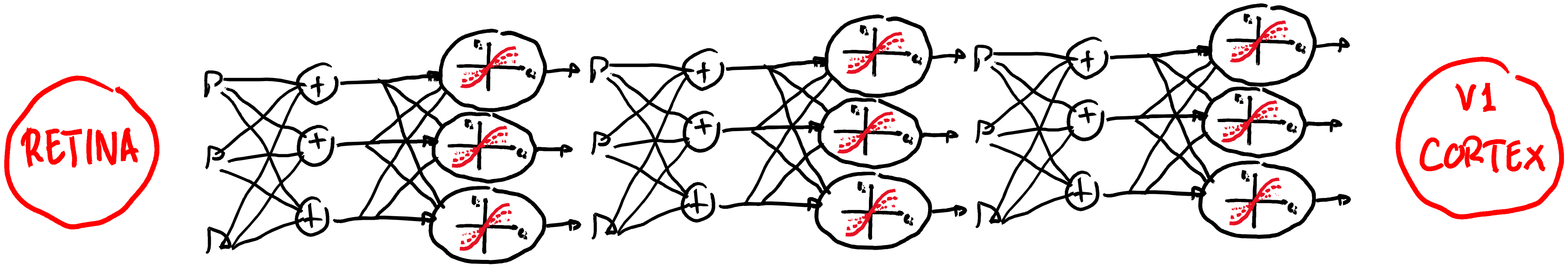
② THE PROBLEM: Quantifying information flow



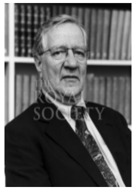
THE EFFICIENT CODING HYPOTHESIS

(information maximization)

Barlow 59, Barlow 01



2 THE PROBLEM: Quantifying information flow



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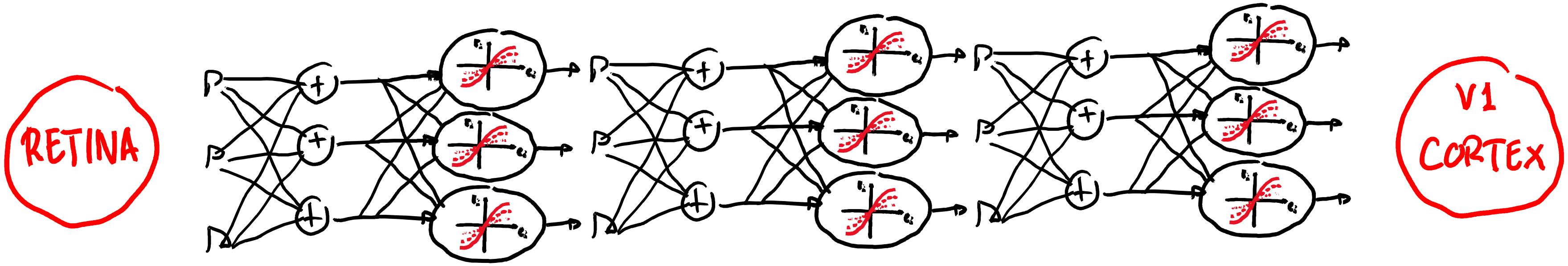


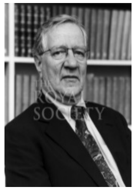
$$x \xrightarrow{S} y = S(x) + n$$

$$I(x,y) = \underbrace{\sum_i h(y_i)}_{(1)} - \underbrace{T(y)}_{(2)} - h(n) \Rightarrow \begin{cases} (1) \text{ MAXIMIZE ENTROPY} \\ (2) \text{ MINIMIZE REDUNDANCY} \end{cases}$$

J. Malo (2020) Spatio-chromatic information available from different neural layers under review **J. Math. Neurosci.** https://arxiv.org/abs/1910.01559

(2) THE PROBLEM: Quantifying information flow

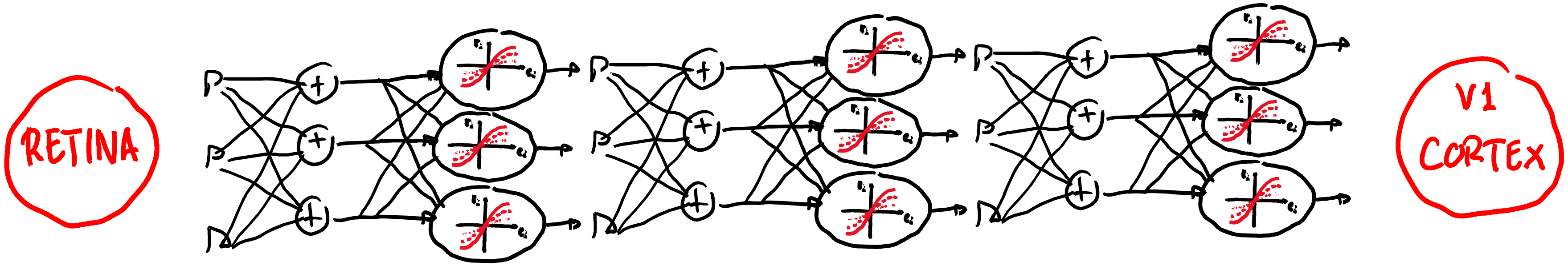
Olshausen & Field 96
Schwartz & Simoncelli 01



THE EFFICIENT CODING HYPOTHESIS
(information maximization)
Barlow 59, Barlow 01

- Standard — STATS → BIOLOGY
- Alternative — BIOLOGY → STATS

Malo & Laparra 10



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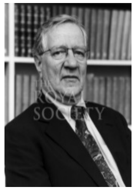
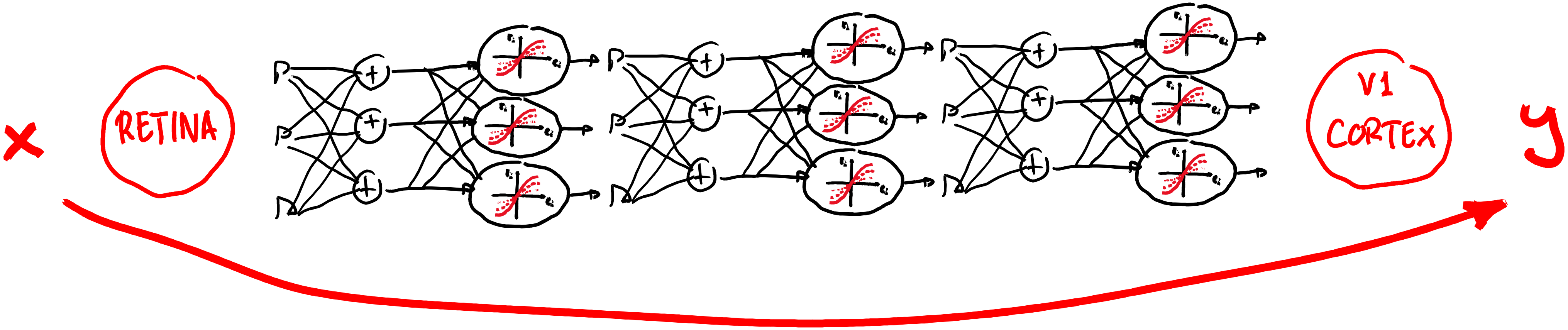


THE EFFICIENT CODING HYPOTHESIS

- Standard STATS → BIOLOGY
- Alternative BIOLOGY → STATS



PROBLEM: ESTIMATING $I(x,y)$, $T(y)$ FROM SAMPLES ≡ **THE CURSE OF DIMENSIONALITY**

3 THE PROPOSED TECHNIQUE : ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

3 THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

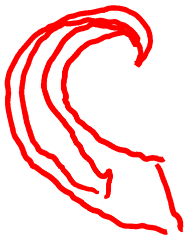
Laparra, Camps, Malo IEEE TNN 2011

Johnson, Laparra, Malo ICML 2019

https://isp.uv.es/RBIG4IT.htm

③ THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

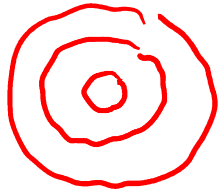
$x^{(0)}$



ANY PDF

$P(x^{(0)})$

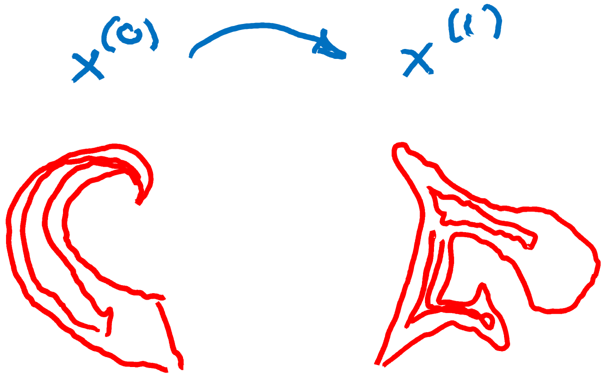
$x^{(N)}$



GAUSSIAN PDF

$P(x^{(N)}) = \mathcal{N}(x^{(N)}, 0, I)$

3 THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)



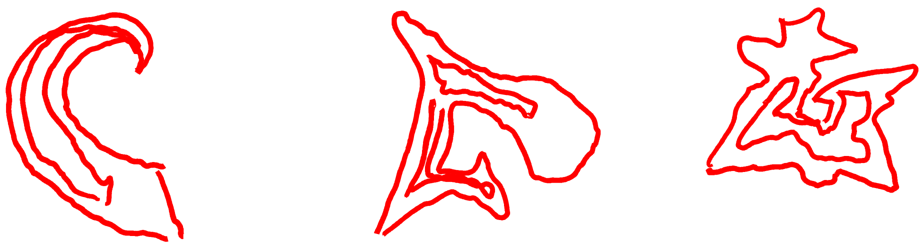
ANY PDF

$p(x^{(0)})$

$$x^{(n+1)} = \underbrace{R^{(n)}}_{\text{Rotation}} \cdot \underbrace{\Psi^{(n)}(x^{(n)})}_{\text{Marginal Gaussianization}}$$

③ THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

$$x^{(0)} \longrightarrow x^{(1)} \longrightarrow x^{(2)}$$



ANY PDF

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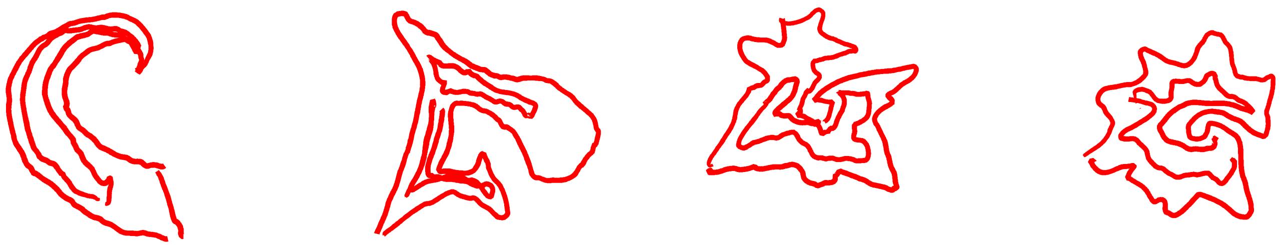
3 THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

$$x^{(0)} \longrightarrow x^{(1)} \longrightarrow x^{(2)} \longrightarrow x^{(3)}$$



ANY PDF

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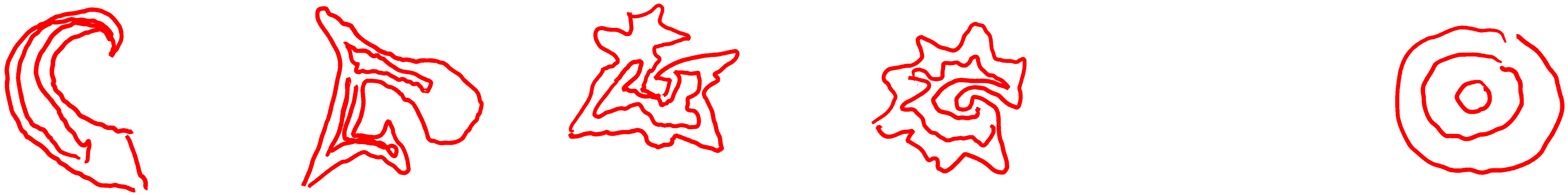
③ THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

$$x^{(0)} \to x^{(1)} \to x^{(2)} \to x^{(3)} \cdots \to x^{(N)}$$



ANY PDF

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GAUSSIAN PDF

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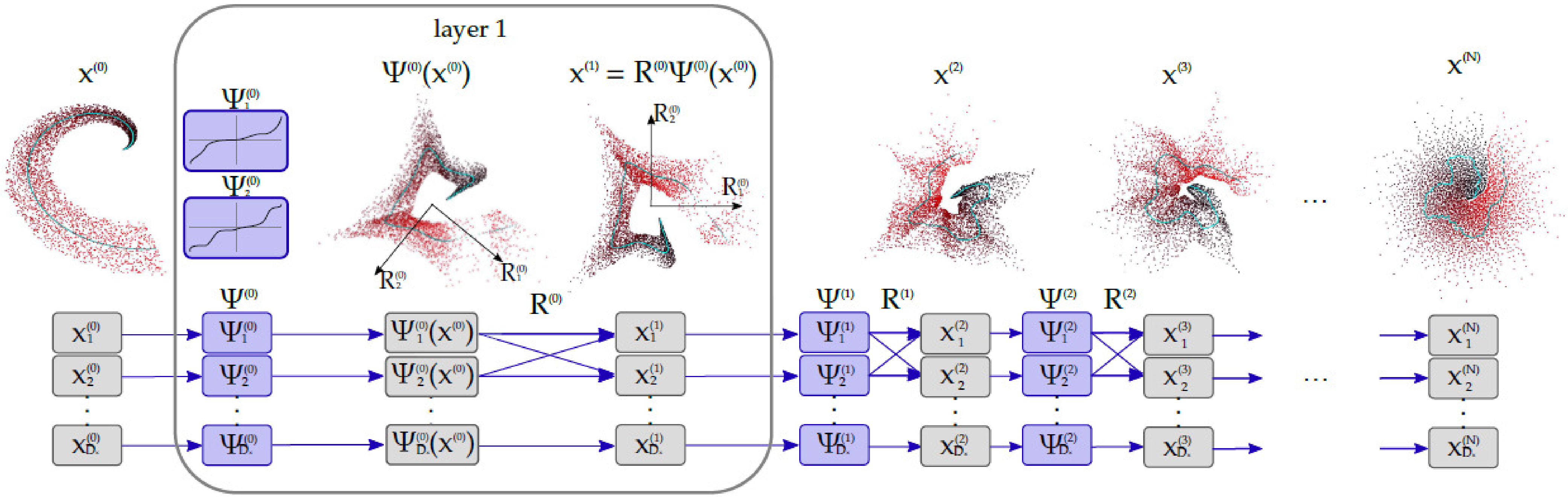

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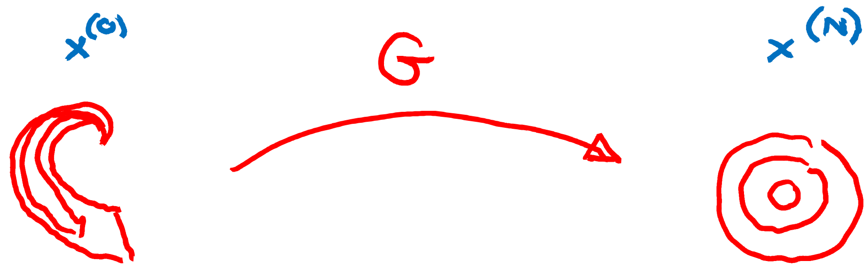


ANY PDF

$p(x^{(0)})$

GAUSSIAN PDF

$p(x^{(N)}) = \mathcal{N}(x^{(N)}, 0, I)$

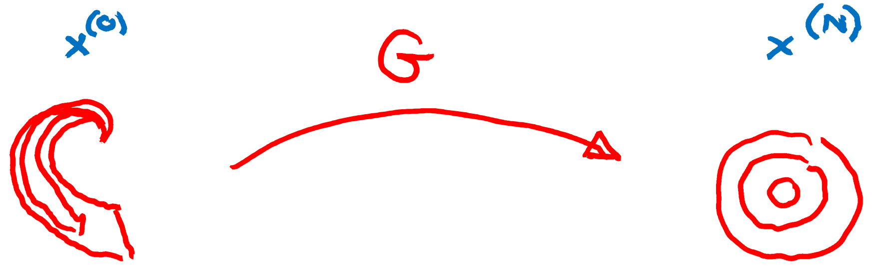
In ANY differentiable transform $\Rightarrow$ In ANY Gaussianization $T(x') = 0$

$$\Delta T(\mathrm{x},\mathrm{x}') = T(\mathrm{x}) - T(\mathrm{x}')$$

$$= \sum_{i=1}^{D_x} H(x_i) - \sum_{i=1}^{D_{x'}} H(x_i') + \frac{1}{2}\underbrace{\mathbb{E}_x\left(\log|\nabla G_x(\mathrm{x})^\top \cdot \nabla G_x(\mathrm{x})|\right)}$$

$$T(\mathrm{x}) = \sum_{i=1}^{D_x} H(x_i) - \frac{D_x}{2}\log(2\pi e) + \underbrace{\mathbb{E}_x\left(\log|\nabla G_x(\mathrm{x})|\right)}$$

③ THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)



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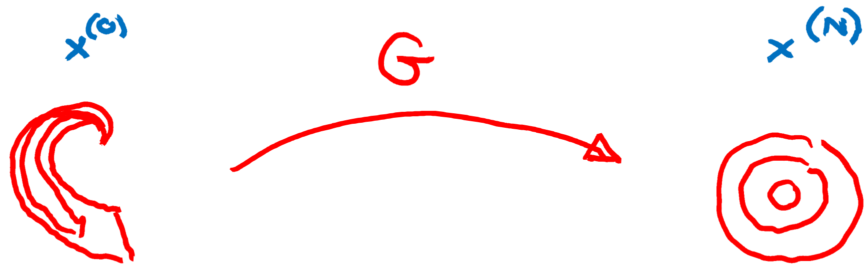
IN RBIG ≡ ONLY UNIVARIATE OPERATIONS

$$\boxed{\tilde{T}(\mathrm{x}) = \sum_{n=0}^{N-1} \Delta \tilde{T}^{(n)} = \frac{(N-1)D_x}{2}\log(2\pi e) - \sum_{n=1}^{N}\sum_{i=1}^{D_x} \tilde{H}(x_i^{(n)})}$$

③ THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)



ANY PDF

$p(x^{(0)})$

GAUSSIAN PDF

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In ANY differentiable transform $\Rightarrow$ In ANY Gaussianization $T(x') = 0$

$$\begin{aligned}\Delta T(\mathrm{x},\mathrm{x}') &= T(\mathrm{x}) - T(\mathrm{x}') \\ &= \sum_{i=1}^{D_x} H(x_i) - \sum_{i=1}^{D_{x'}} H(x'_i) + \frac{1}{2}\mathbb{E}_x\Big(\underbrace{\log|\nabla G_x(\mathrm{x})^\top \cdot \nabla G_x(\mathrm{x})|}\Big)\end{aligned}$$

$$T(\mathrm{x}) = \sum_{i=1}^{D_x} H(x_i) - \frac{D_x}{2}\log(2\pi e) + \mathbb{E}_x\Big(\underbrace{\log|\nabla G_x(\mathrm{x})|}\Big)$$

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$\Rightarrow$ CURSE OF DIMENSION ALLEVIATED!

3 THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

Total Correlation

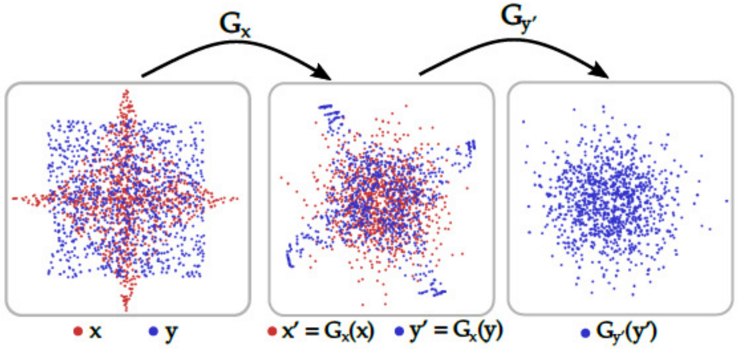
$$\tilde{T}(\mathbf{x}) = \sum_{n=0}^{N-1} \Delta \tilde{T}^{(n)} = \frac{(N-1)D_x}{2}\log(2\pi e) - \sum_{n=1}^{N}\sum_{i=1}^{D_x} \tilde{H}(x_i^{(n)})$$

Differential Entropy

$$\tilde{H}(\mathbf{x}) = \sum_{i=1}^{D_x} \tilde{H}(x_i) - \tilde{T}(\mathbf{x})$$

Kullback-Leibler Div.

$$D_{\mathrm{KL}}(\mathbf{y}|\mathbf{x}) = D_{\mathrm{KL}}(G_x(\mathbf{y})|G_x(\mathbf{x})) = T(\mathbf{x}) + \sum_{i=1}^{D_x} D_{\mathrm{KL}}(p_{x_i}(x_i)|\mathcal{N}(0,1))$$



Mutual Information

$$\tilde{I}(\mathbf{x},\mathbf{y}) = \tilde{T}([G_x(\mathbf{x}), G_y(\mathbf{y})])$$

③ THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

Total Correlation

$\tilde{T}(\mathbf{x})$

		D_x	RBIG	kNN	KDP	expF	vME	Ens
Gaussian	-	3	0.87	0.94	76.65	0.63	4.27	4.03
		10	0.97	23.48	>100	0.27	31.72	34.83
		50	1.45	45.77	>100	0.52	>100	54.74
		100	1.55	52.78	>100	0.41	>100	59.94
Rotated	-	3	1.70	1.80	82.90	16.80	1.90	9.40
		10	8.30	27.20	>100	11.00	24.20	38.70
		50	7.70	51.10	>100	15.10	>100	59.40
		100	7.50	57.80	>100	15.50	>100	64.50
Student	$\nu = 3$	3	7.01	13.55	>100	94.03	>100	66.59
		10	32.93	16.73	>100	67.32	>100	15.27
		50	18.18	12.02	>100	29.44	>100	24.65
		100	12.71	17.41	>100	21.12	>100	28.63
	$\nu = 5$	3	26.61	52.76	>100	89.74	81.85	133.12
		10	23.94	19.74	>100	49.60	>100	12.31
		50	10.10	16.87	>100	20.29	>100	32.14
		100	7.10	22.53	>100	15.39	>100	34.96
	$\nu = 20$	3	88.27	>100	>100	48.56	>100	>100
		10	3.05	11.86	>100	10.51	>100	19.93
		50	3.07	33.17	>100	4.54	>100	52.62
		100	1.31	35.56	>100	3.43	>100	49.46

Differential Entropy

$\tilde{H}(\mathbf{x})$

		D_x	RBIG	kNN	KDP	expF	vME	Ens
Gaussian		3	0.90	1.70	112.30	1.10	8.80	12.00
		10	1.20	27.90	179.80	0.10	34.70	40.30
		50	0.90	32.20	107.40	0.10	108.40	38.10
		100	0.70	30.70	89.60	0.10	94.20	34.60
Rotated		3	4.00	6.20	171.40	36.80	3.70	30.10
		10	12.20	38.50	241.80	17.90	31.80	53.90
		50	6.80	44.60	136.60	13.50	87.90	51.60
		100	5.50	42.50	110.70	11.50	94.30	47.20
Student	$\nu = 3$	3	0.75	0.76	34.93	11.90	3.13	1.99
		10	2.82	1.44	137.19	15.55	53.19	1.77
		50	6.03	3.47	195.30	22.11	175.94	7.09
		100	6.61	8.57	228.62	24.44	166.09	13.72
	$\nu = 5$	3	0.51	0.70	24.75	3.42	1.38	1.94
		10	1.12	1.25	96.73	5.52	59.29	1.21
		50	2.82	4.84	146.63	9.59	202.48	8.89
		100	3.11	10.67	184.02	11.36	195.17	16.24
	$\nu = 20$	3	0.54	0.71	19.19	0.76	1.32	1.56
		10	0.48	0.84	69.83	1.56	46.62	0.37
		50	0.95	6.66	107.74	3.31	219.86	11.13
		100	0.68	13.45	138.98	4.20	214.41	19.35

Kullback-Leibler Div.

$D_{\mathrm{KL}}(\mathbf{y}|\mathbf{x})$

		dim	RBIG	kNN	expF	vME
Gaussian, different means	$\mu_2 = 0.2$	3	13.49	16.90	10.28	92.28
		10	19.50	22.47	3.27	>1000
		50	32.04	40.85	13.34	>1000
		100	47.40	41.24	28.14	>1000
	$\mu_2 = 0.4$	3	3.55	7.98	5.57	24.22
		10	5.25	22.93	2.02	604.17
		50	8.67	40.91	3.40	>1000
		100	4.83	43.49	8.70	>1000
	$\mu_2 = 0.6$	3	3.75	6.39	3.89	12.23
		10	2.81	24.72	1.85	213.72
		50	13.83	43.11	1.83	897.65
		100	42.42	46.00	5.11	686.96
Gaussians, different covs.	$\mu_2 = 0.5$	3	24.93	27.30	4.89	63.90
		10	18.80	103.65	2.64	>1000
		50	23.62	173.62	8.42	>1000
		100	32.56	200.33	17.59	>1000
	$\mu_2 = 0.75$	3	21.04	24.77	3.72	36.64
		10	10.44	96.85	1.86	605.00
		50	10.07	159.16	5.70	>1000
		100	13.66	179.67	11.40	>1000
	$\mu_2 = 0.9$	3	17.12	25.95	3.40	26.15
		10	6.77	94.42	1.60	448.87
		50	3.40	152.46	4.81	>1000
		100	5.96	170.28	9.43	>1000
Gaussian vs. Student	$\nu = 2$	3	5.08	29.58	793.53	5.78
		10	32.72	83.51	1278.91	596.63
		50	59.37	468.33	2783.43	>1000
		100	42.11	1024.30	4330.18	>1000
	$\nu = 4$	3	17.09	95.02	148.08	22.52
		10	42.84	157.63	219.26	963.37
		50	60.53	584.46	547.48	>1000
		100	41.71	1214.61	962.45	>1000
	$\nu = 7$	3	8.34	271.61	35.78	59.69
		10	38.78	307.82	49.77	>1000
		50	48.80	713.36	145.15	>1000
		100	26.01	1399.34	278.93	>1000
Student vs. Student	$\nu = 2$	3	9.08	13.87	3442.45	>1000
		10	20.57	57.60	7462.58	346.61
		50	85.14	405.47	19991.36	>1000
		100	242.80	939.24	35064.60	>1000
	$\nu = 4$	3	9.51	47.03	1502.19	48.89
		10	36.33	139.12	2561.86	>1000
		50	37.29	656.95	7997.12	>1000
		100	60.52	1441.18	13033.03	>1000
	$\nu = 7$	3	13.13	126.41	589.47	128.84
		10	23.13	301.97	1070.70	>1000
		50	28.34	976.95	3689.57	>1000
		100	145.88	2046.95	6370.43	>10000

Mutual Information

$\tilde{I}(\mathbf{x}, \mathbf{y})$

		D_x	RBIG	kNN	KDP	expF	vME	Ens
Gaussians		3	10.60	26.00	149.10	9.20	13.20	48.50
		10	9.60	76.30	102.60	23.70	311.00	91.00
		50	6.80	104.70	100.70	39.50	68.00	105.50
		100	11.70	107.20	>1000	42.60	77.40	106.10
Student vs. Student	$\nu = 3$	3	35.72	95.32	>1000	63.73	>1000	86.58
		10	22.26	2.66	118.51	18.14	>1000	66.77
		50	1.51	88.38	104.50	36.10	810.02	105.83
		100	15.34	98.66	>1000	65.71	789.55	105.34
	$\nu = 5$	3	18.51	118.04	>1000	56.49	>1000	96.41
		10	3.07	24.83	113.89	9.39	>1000	101.26
		50	10.91	102.89	105.08	25.17	849.12	117.30
		100	24.43	105.41	101.10	42.57	805.44	110.58
	$\nu = 20$	3	73.63	194.16	>1000	14.63	>1000	15.36
		10	40.02	108.82	110.68	29.69	>1000	208.20
		50	29.98	149.53	102.93	36.30	946.93	154.88
		100	37.21	128.27	101.44	43.77	844.41	127.67

Szabo JMLR 2014
- KNN
- Partition trees
- Exp. Family
- Von Mises
- Ensemble

https://isp.uv.es/RBIG4IT.htm

(3) THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

Total Correlation

$\tilde{T}(\mathbf{x})$

		D_x	RBIG	kNN	KDP	expF	vME	Ens
Gaussian	-	3	0.87	0.94	76.65	0.63	4.27	4.03
		10	0.97	23.48	>100	0.27	31.72	34.83
		50	1.45	45.77	>100	0.52	>100	54.74
		100	1.55	52.78	>100	0.41	>100	59.94
Rotated	-	3	1.70	1.80	82.90	16.80	1.90	9.40
		10	8.30	27.20	>100	11.00	24.20	38.70
		50	7.70	51.10	>100	15.10	>100	59.40
		100	7.50	57.80	>100	15.50	>100	64.50
Student	$\nu = 3$	3	7.01	13.55	>100	94.03	>100	66.59
		10	32.93	16.73	>100	67.32	>100	15.27
		50	18.18	12.02	>100	29.44	>100	24.65
		100	12.71	17.41	>100	21.12	>100	28.63
	$\nu = 5$	3	26.61	52.76	>100	89.74	81.85	133.12
		10	23.94	19.74	>100	49.60	>100	12.31
		50	10.10	16.87	>100	20.29	>100	32.14
		100	7.10	22.53	>100	15.39	>100	34.96
	$\nu = 20$	3	88.27	>100	>100	48.56	>100	>100
		10	3.05	11.86	>100	10.51	>100	19.93
		50	3.07	33.17	>100	4.54	>100	52.62
		100	1.31	35.56	>100	3.43	>100	49.46

Differential Entropy

$\tilde{H}(\mathbf{x})$

		D_x	RBIG	kNN	KDP	expF	vME	Ens
Gaussian		3	0.90	1.70	112.30	1.10	8.80	12.00
		10	1.20	27.90	179.80	0.10	34.70	40.30
		50	0.90	32.20	107.40	0.10	108.40	38.10
		100	0.70	30.70	89.60	0.10	94.20	34.60
Rotated		3	4.00	6.20	171.40	36.80	3.70	30.10
		10	12.20	38.50	241.80	17.90	31.80	53.90
		50	6.80	44.60	136.60	13.50	87.90	51.60
		100	5.50	42.50	110.70	11.50	94.30	47.20
Student	$\nu = 3$	3	0.75	0.76	34.93	11.90	3.13	1.99
		10	2.82	1.44	137.19	15.55	53.19	1.77
		50	6.03	3.47	195.30	22.11	175.94	7.09
		100	6.61	8.57	228.62	24.44	166.09	13.72
	$\nu = 5$	3	0.51	0.70	24.75	3.42	1.38	1.94
		10	1.12	1.25	96.73	5.52	59.29	1.21
		50	2.82	4.84	146.63	9.59	202.48	8.89
		100	3.11	10.67	184.02	11.36	195.17	16.24
	$\nu = 20$	3	0.54	0.71	19.19	0.76	1.32	1.56
		10	0.48	0.84	69.83	1.56	46.62	0.37
		50	0.95	6.66	107.74	3.31	219.86	11.13
		100	0.68	13.45	138.98	4.20	214.41	19.35

Kullback-Leibler Div.

$D_{\mathrm{KL}}(\mathbf{y}|\mathbf{x})$

		dim	RBIG	kNN	expF	vME
Gaussian, different means	$\mu_2 = 0.2$	3	13.49	16.90	10.28	92.28
		10	19.50	22.47	3.27	>1000
		50	32.04	40.85	13.34	>1000
		100	47.40	41.24	28.14	>1000
	$\mu_2 = 0.4$	3	3.55	7.98	5.57	24.22
		10	5.25	22.93	2.02	604.17
		50	8.67	40.91	3.40	>1000
		100	4.83	43.49	8.70	>1000
	$\mu_2 = 0.6$	3	3.75	6.39	3.89	12.23
		10	2.81	24.72	1.85	213.72
		50	13.83	43.11	1.83	897.65
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		50	3.40	152.46	4.81	>1000
		100	5.96	170.28	9.43	>1000
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	$\nu = 4$	3	17.09	95.02	148.08	22.52
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		50	85.14	405.47	19991.36	>1000
		100	242.80	939.24	35064.60	>1000
	$\nu = 4$	3	9.51	47.03	1502.19	48.89
		10	36.33	139.12	2561.86	>1000
		50	37.29	656.95	7997.12	>1000
		100	60.52	1441.18	13033.03	>1000
	$\nu = 7$	3	13.13	126.41	589.47	128.84
		10	23.13	301.97	1070.70	>1000
		50	28.34	976.95	3689.57	>1000
		100	145.88	2046.95	6370.43	>10000

Szabo JMLR 2014
- KNN
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- Ensemble

https://isp.uv.es/RBIG4IT.htm

Mutual Information

$\tilde{I}(\mathbf{x}, \mathbf{y})$

		D_x	RBIG	kNN	KDP	expF	vME	Ens
Gaussians		3	10.60	26.00	149.10	9.20	13.20	48.50
		10	9.60	76.30	102.60	23.70	311.00	91.00
		50	6.80	104.70	100.70	39.50	68.00	105.50
		100	11.70	107.20	>1000	42.60	77.40	106.10
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		10	22.26	2.66	118.51	18.14	>1000	66.77
		50	1.51	88.38	104.50	36.10	810.02	105.83
		100	15.34	98.66	>1000	65.71	789.55	105.34
	$\nu = 5$	3	18.51	118.04	>1000	56.49	>1000	96.41
		10	3.07	24.83	113.89	9.39	>1000	101.26
		50	10.91	102.89	105.08	25.17	849.12	117.30
		100	24.43	105.41	101.10	42.57	805.44	110.58
	$\nu = 20$	3	73.63	194.16	>1000	14.63	>1000	15.36
		10	40.02	108.82	110.68	29.69	>1000	208.20
		50	29.98	149.53	102.93	36.30	946.93	154.88
		100	37.21	128.27	101.44	43.77	844.41	127.67

RBIG Info-theory measures work!

4 EXPERIMENTS & RESULTS

4 EXPERIMENTS & RESULTS

Materials

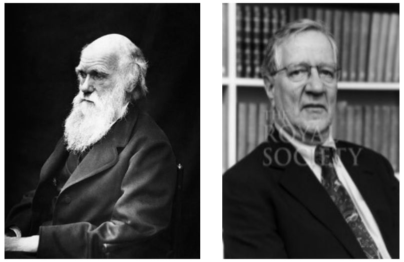
- Natural Images
- Vision model : Retina - Cortex { - Performance
  - Assumptions

Experiments Efficient Coding

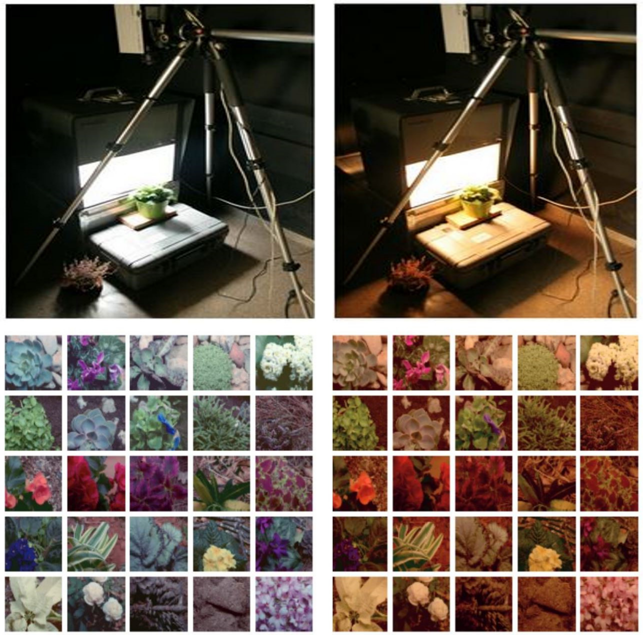
- Global { - $\Delta T(x,y)$
  - $I(x,y)$ | Layers & flexibility
- Local { - $\Delta T(x,y)$
  - $I(x,y)$ | PDF

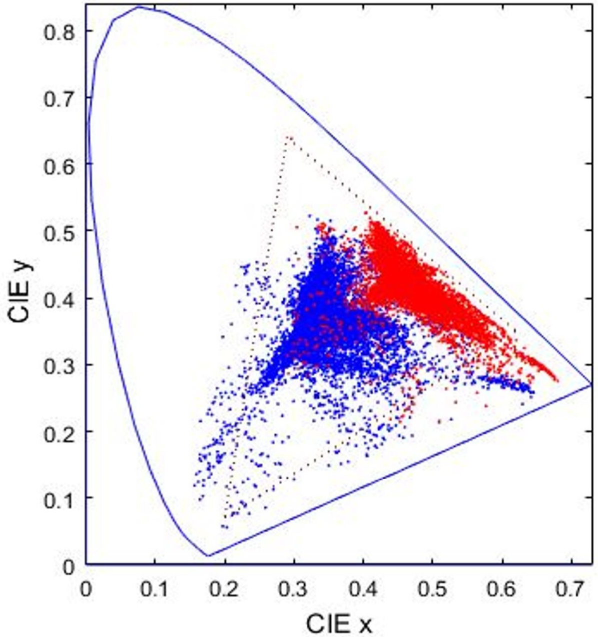
Experiments Image Quality

④ EXPERIMENTS & RESULTS Material: Natural images



Natural Environment



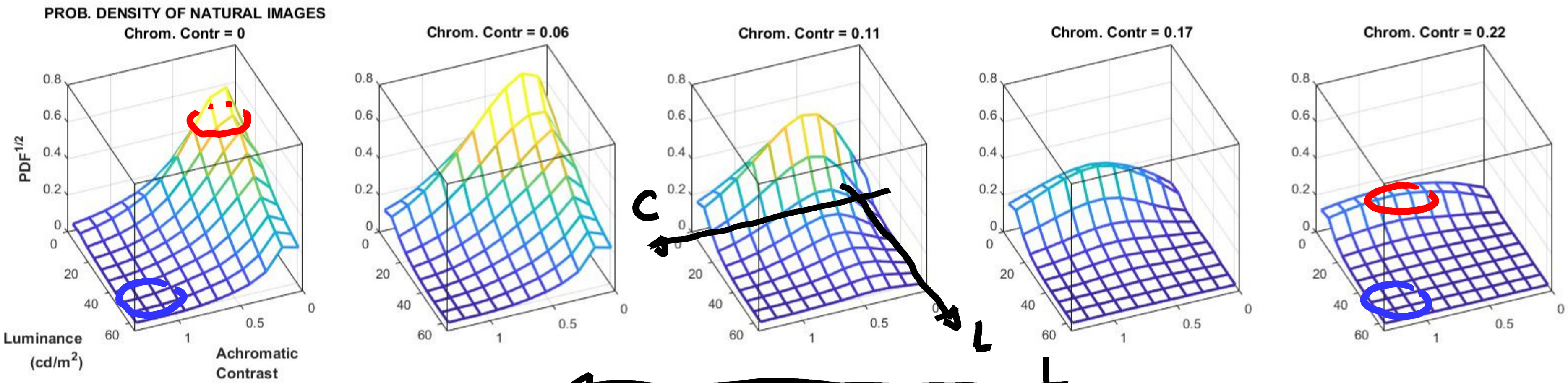


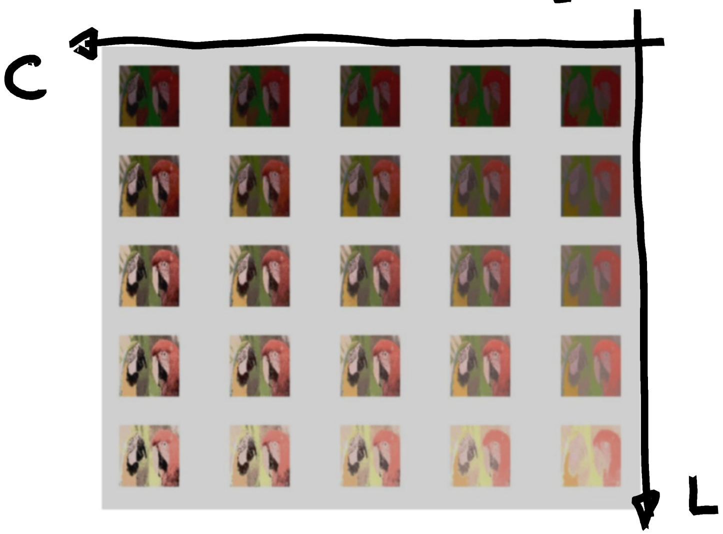
Laparra & Malo **Neural Comp.** 2012, Gutmann & Malo **PLOS** 2014 https://isp.uv.es/data_color.htm

Gómez & Malo **J. Neurophysiol.** 2020 https://isp.uv.es/code/visioncolor/infoWilsonCowan.html

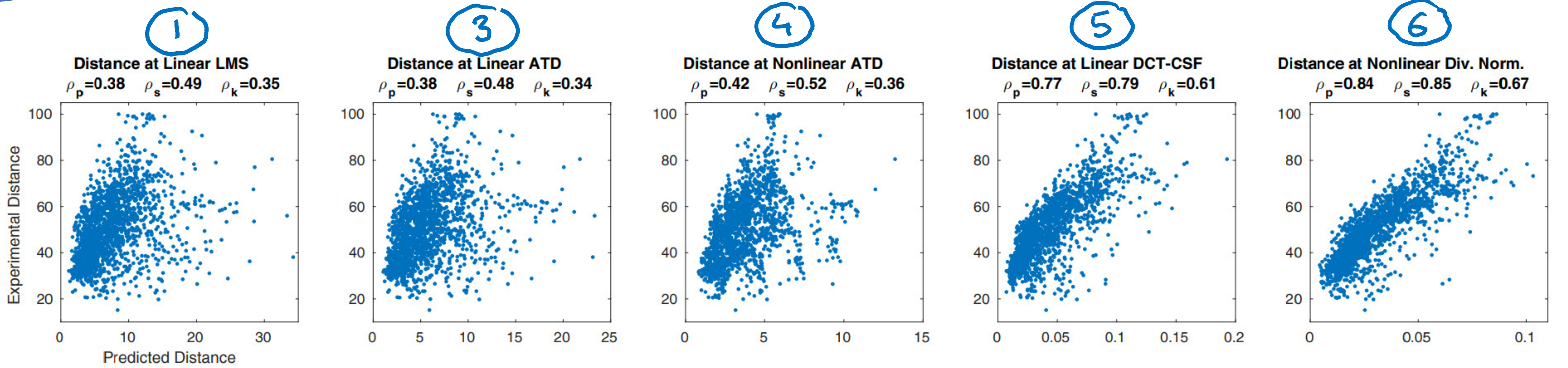
4 EXPERIMENTS & RESULTS Material: Natural images

Probability Density Function of Natural Images



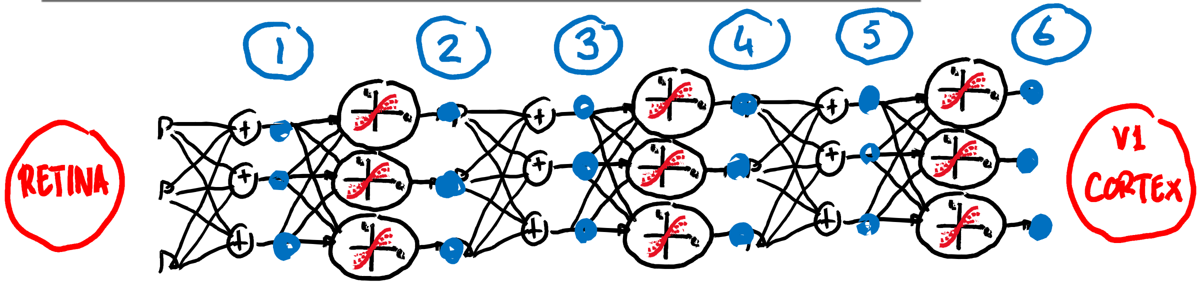


4 EXPERIMENTS & RESULTS   Material: Vision Model

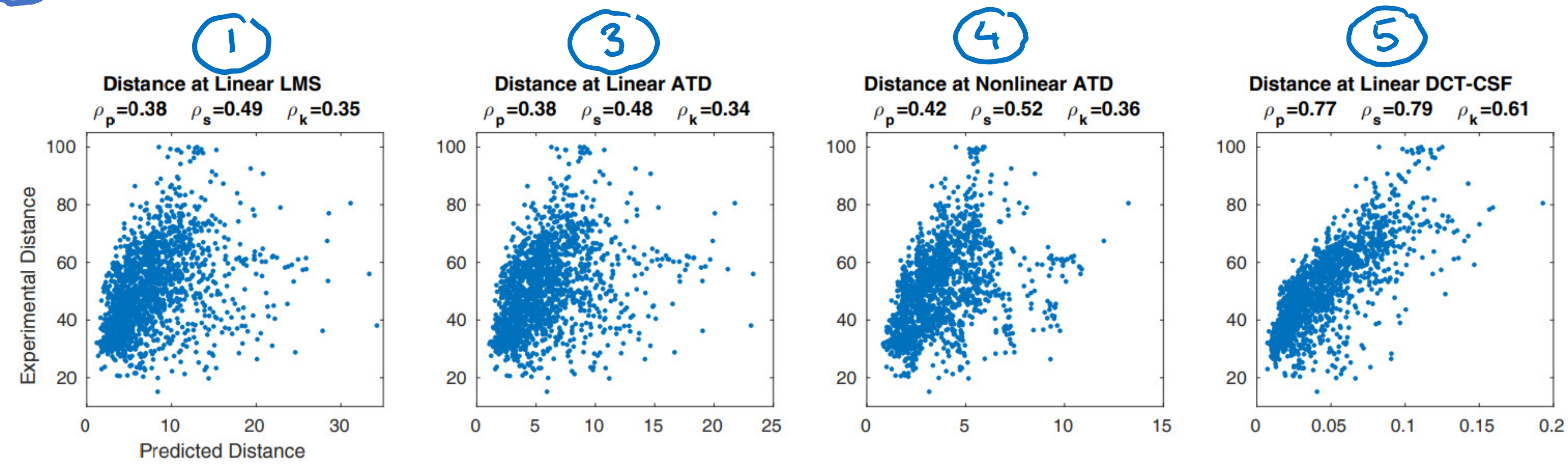


Pearson correlation with human viewers using different building blocks (or model layers)

	Spatial Extent	$r^{(1)}$	$r^{(2)}$	$x^{(2)}$	$r^{(3)}$	$x^{(3)}$
More flexible model	(0.27 deg)	0.38	0.38	0.42	0.77	0.51
Baseline model	(0.27 deg)	0.38	0.38	0.42	0.77	**0.84**
More rigid model	(0.27 deg)	0.38	0.38	0.42	0.77	0.79
Totally rigid model	(0.27 deg)	0.38	0.38	0.38	0.68	0.68
Baseline model	(0.05 deg)	0.26	0.27	0.31	0.37	0.40



4 EXPERIMENTS & RESULTS Material: Vision Model



Pearson correlation with human viewers using different building blocks (or model layers)

	Spatial Extent	$r^{(1)}$	$r^{(2)}$	$x^{(2)}$	$r^{(3)}$	$x^{(3)}$
More flexible model	(0.27 deg)	0.38	0.38	0.42	0.77	0.51
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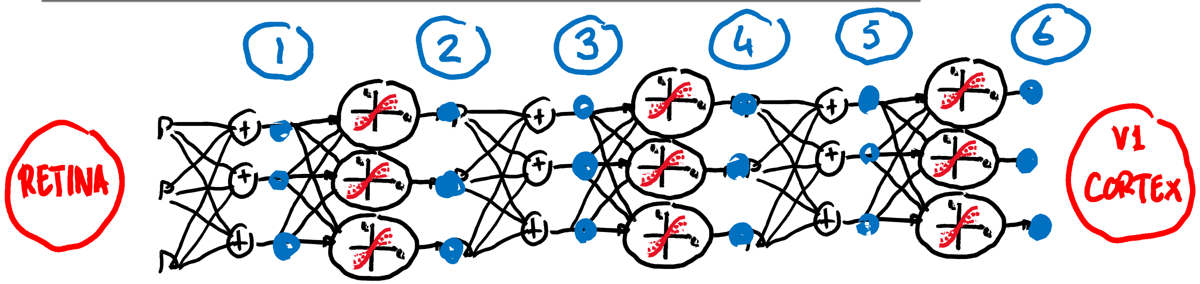

Assumptions:

(1) single-step transforms

$x \longrightarrow y \longrightarrow z \longrightarrow k$

(2) Constant SNR (5% noise)

Efficient Coding (global)

Redundancy Reduction

$\Delta T(x^{input}, x^{resp.})$

Pearson correlation with human viewers using different building blocks (or model layers).

	Spatial Extent	$\boldsymbol{r}^{(1)}$	$\boldsymbol{r}^{(2)}$	$\boldsymbol{x}^{(2)}$	$\boldsymbol{r}^{(3)}$	$\boldsymbol{x}^{(3)}$
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Totally rigid model	(0.27 deg)	0.38	0.38	0.38	0.68	0.68
Baseline model	(0.05 deg)	0.26	0.27	0.31	0.37	0.40

Transmitted Information

$I(x^{in}, x^{resp})$

24/30

4 EXPERIMENTS & RESULTS

Redundancy Reduction

$\Delta T(x^{input}, x^{resp.})$

ΔT_{RBIG}

$$\Delta T_{theor} = \sum_i h(x_i^{in}) - h(x_i^{resp}) + E_x\left[\log_2 |\nabla S|\right]$$

Pearson correlation with human viewers using different building blocks (or model layers)

	Spatial Extent	$r^{(1)}$	$r^{(2)}$	$x^{(2)}$	$r^{(3)}$	$x^{(3)}$
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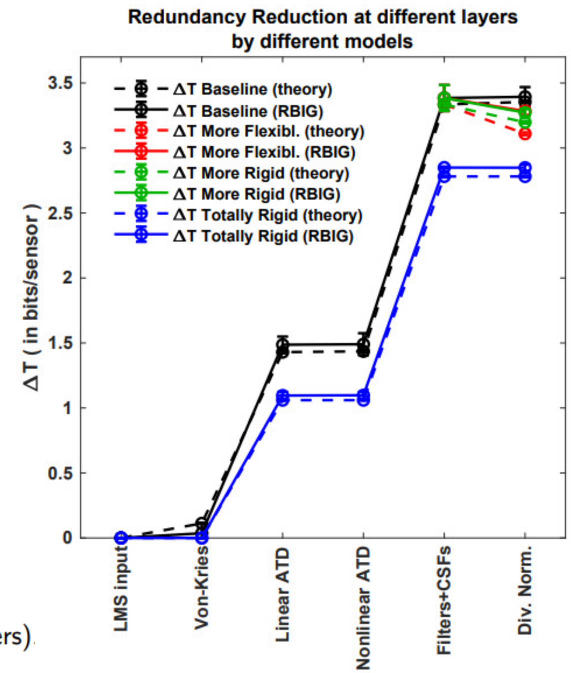

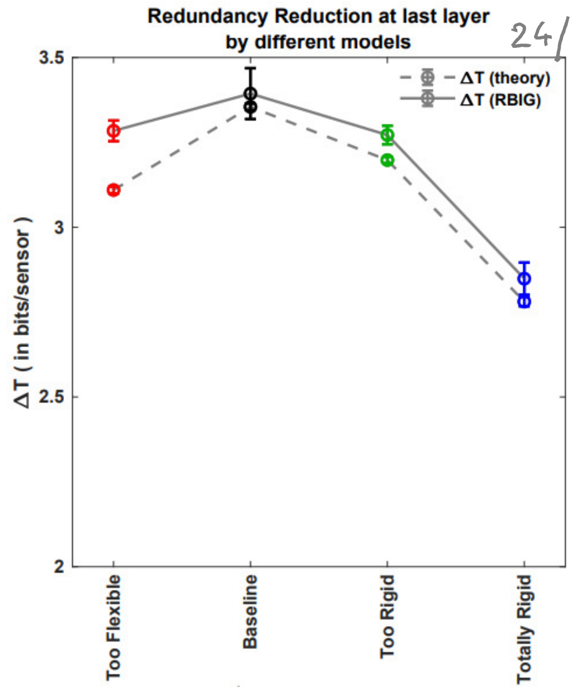

Transmitted Information

$I(x^{in}, x^{resp})$

4 EXPERIMENTS & RESULTS

24/30

Redundancy Reduction

$\Delta T(x^{input}, x^{resp.})$

ΔT_{RBIG}

$$\Delta T_{theor} : \sum_i h(x_i^{in}) - h(x_i^{resp}) + E_x\left[\log_2 |\nabla S|\right]$$

Pearson correlation with human viewers using different building blocks (or model layer

	Spatial Extent	$r^{(1)}$	$r^{(2)}$	$x^{(2)}$	$r^{(3)}$	$x^{(3)}$
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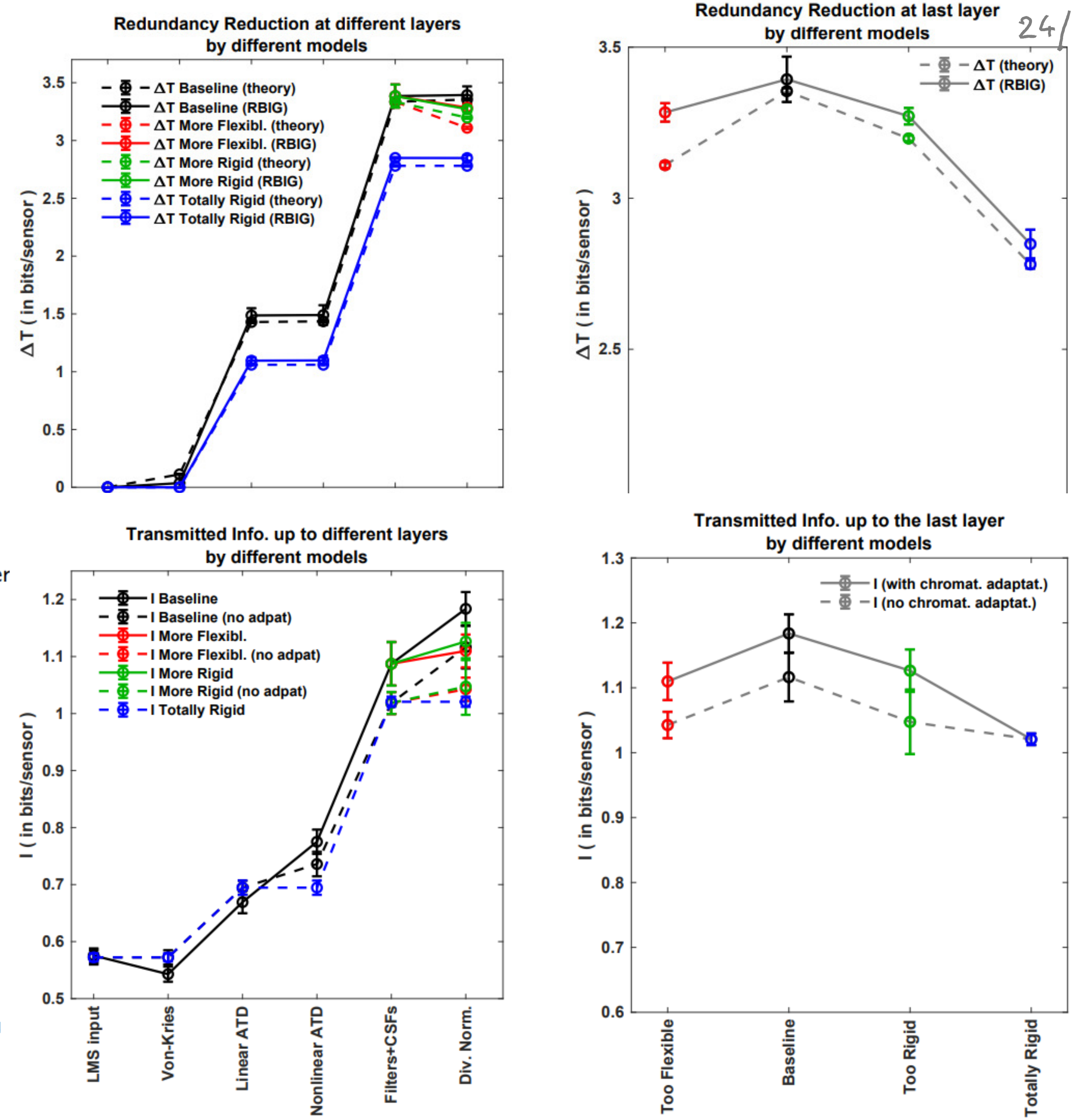
Transmitted Information

$I(x^{in}, x^{resp})$



4 EXPERIMENTS & RESULTS

Redundancy Reduction

$\Delta T(x^{input}, x^{resp})$

ΔT_{RBIG}

$$\Delta T_{theor} = \sum_i h(x_i^{in}) - h(x_i^{resp}) + E_x\left[\log_2 |\nabla S|\right]$$

Pearson correlation with human viewers using different building blocks (or model layer

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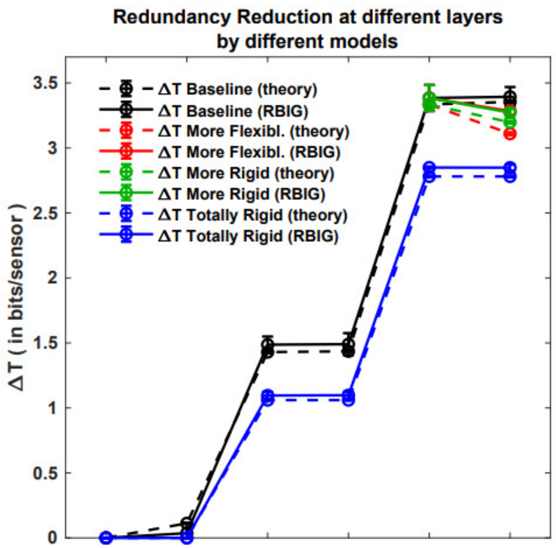
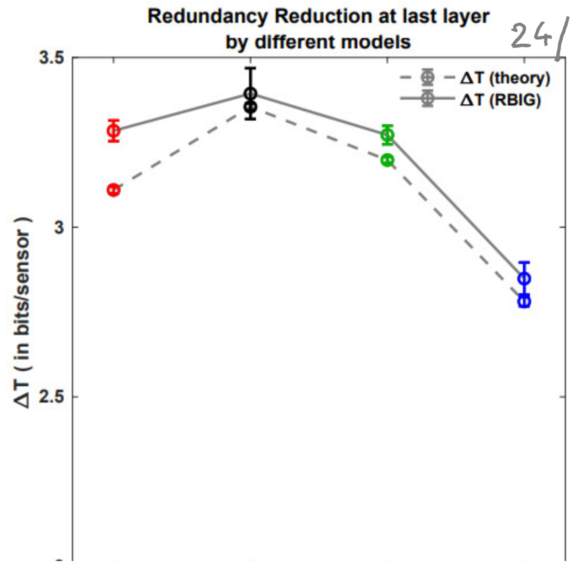
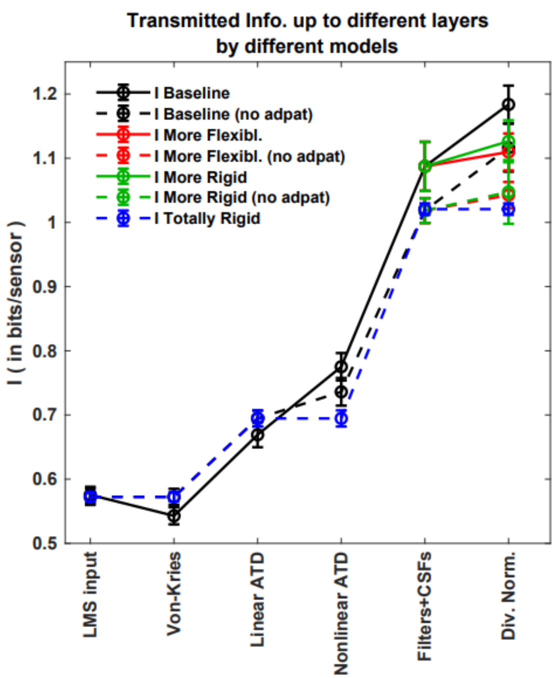
- Deeper is better
- Basline is better
- Space vs Color

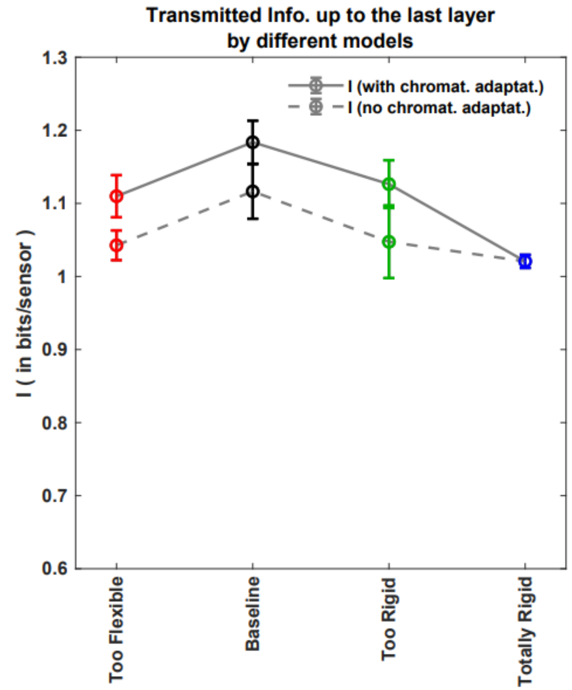
Transmitted Information

$I(x^{in}, x^{resp})$









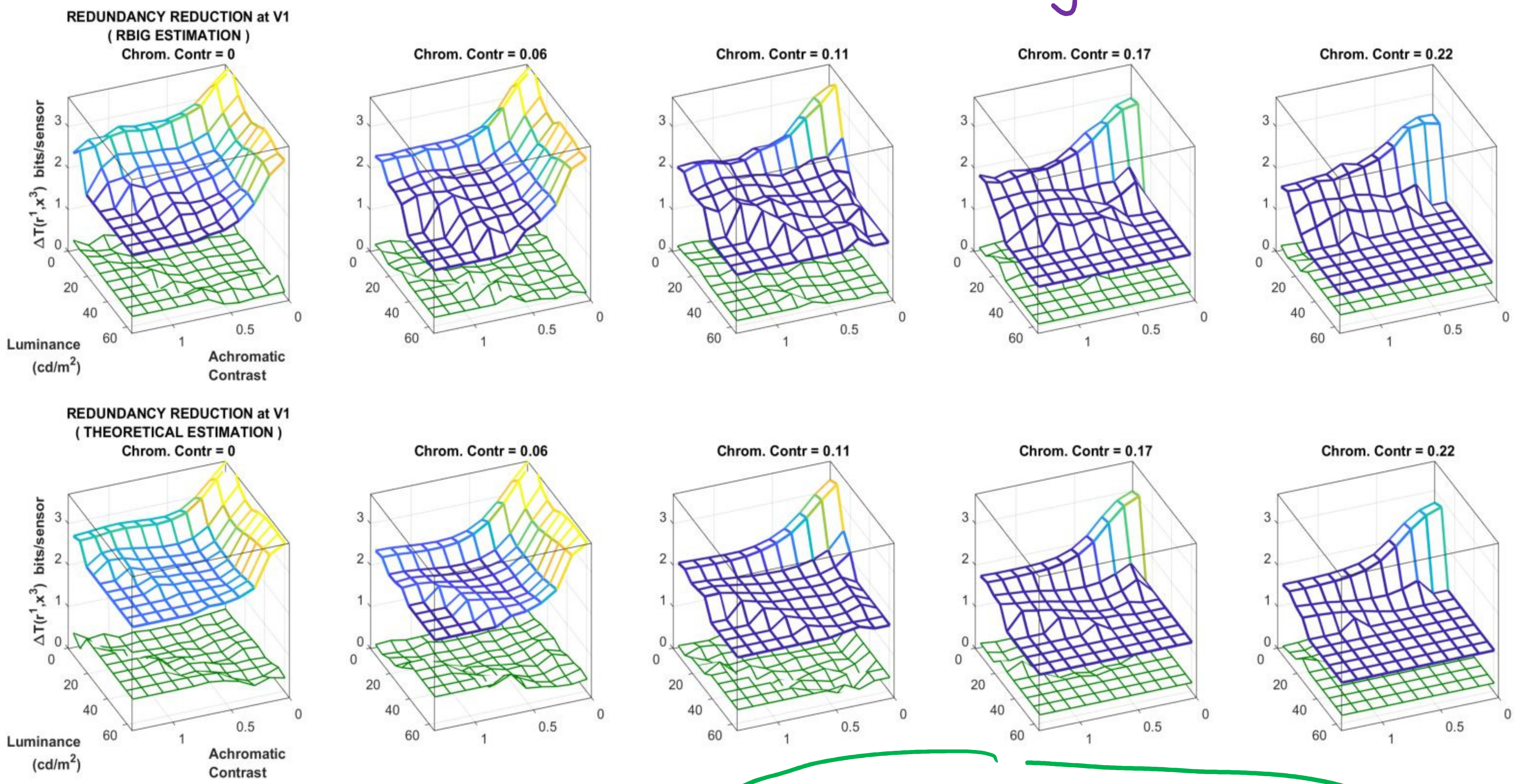
④ EXPERIMENTS & RESULTS

Efficient Coding (local)

Redundancy Reduction

RBIG

THEORY



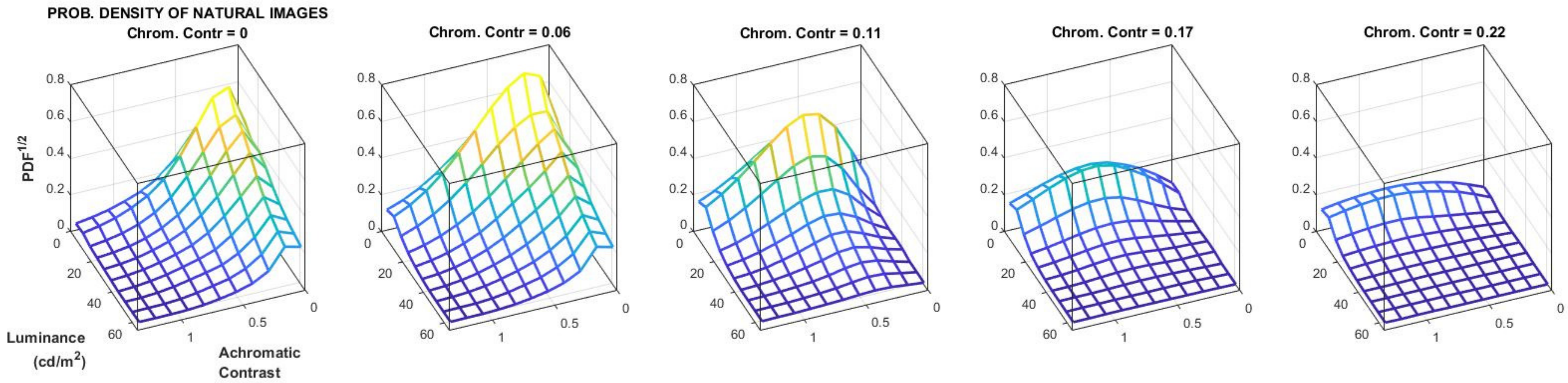
RBIG estimates work!

4 EXPERIMENTS & RESULTS

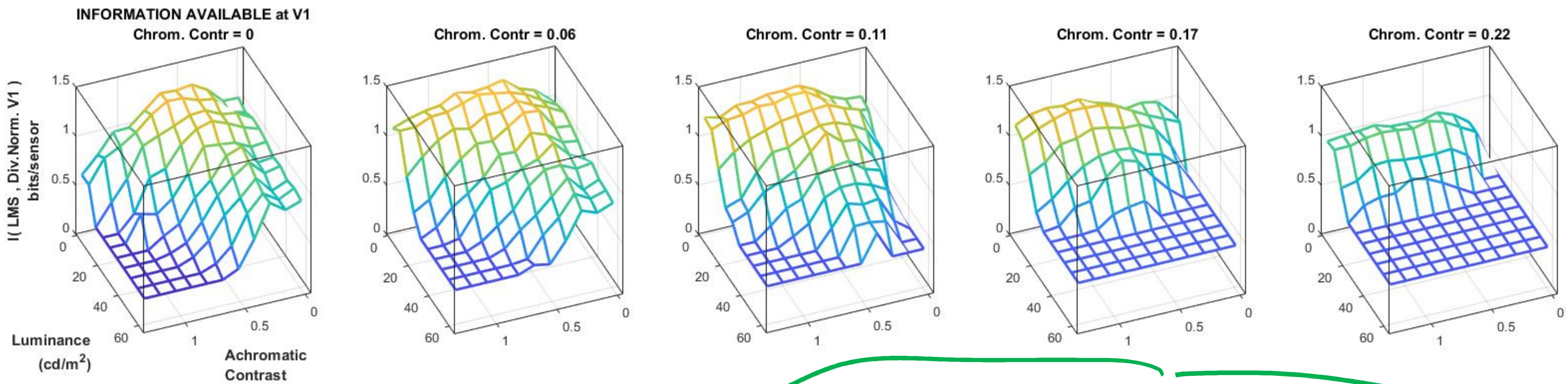
Efficient Coding (local)

Transmitted Information

PDF Natural Images

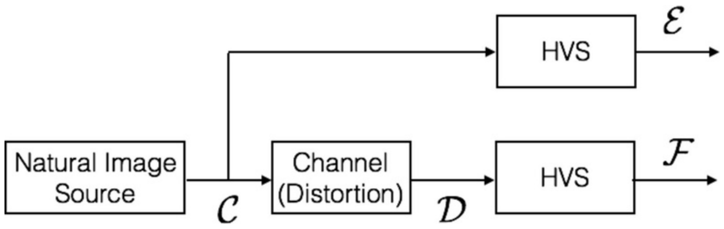


$I(x^{in}, x^{resp})$



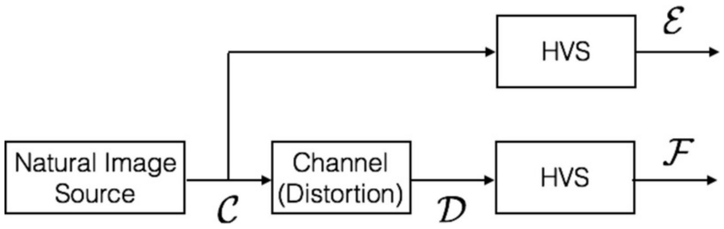
Transmitted info matches PDF

Visual Information Fidelity



$$\mathrm{VIF} = \frac{I(C, S(D))}{I(C, S(C))} = \frac{I(C, F)}{I(C, E)}$$

Visual Information Fidelity



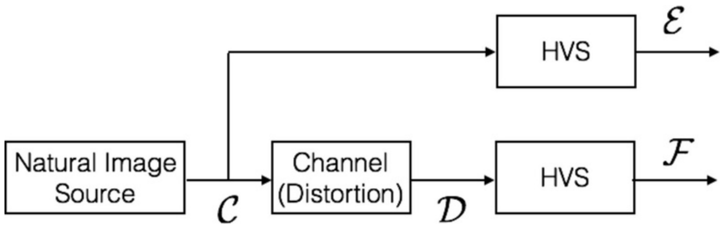
Conventional VIF

- Vision Model: Wavelet + Gauss. Noise
- Mutual Inform. Assumes G.S.M.

$$\mathrm{VIF} = \frac{I(C, S(D))}{I(C, S(C))} = \frac{I(C,F)}{I(C,E)} = \frac{\sum_i \log_2\left(\frac{|g_i s_i C_u + (\sigma_i^2+\sigma_n^2)\mathbb{1}|}{|(\sigma_i^2+\sigma_n^2)\mathbb{1}|}\right)}{\sum_i \log_2\left(\frac{|s_i C_u + \sigma_i^2\mathbb{1}|}{|\sigma_i^2\mathbb{1}|}\right)}$$

4 EXPERIMENTS & RESULTS

Visual Information Fidelity



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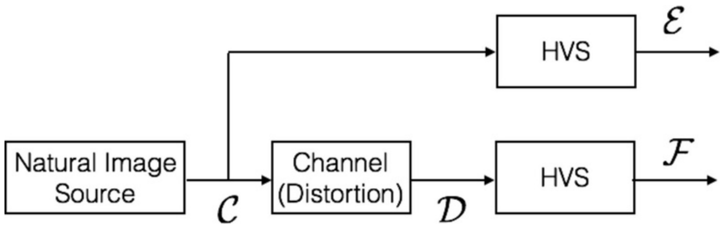
Alternative VIF:

* Vision Model: Wavelet + Gauss. Noise
* Compute $\left\{ \begin{array}{l} I(C,F) \\ I(C,E) \end{array} \right.$ empirically using RBIG

4 EXPERIMENTS & RESULTS

Visual Information Fidelity



Conventional VIF

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$$\mathrm{VIF} = \frac{I(C,S(D))}{I(C,S(C))} = \frac{I(C,F)}{I(C,E)} = \frac{\sum_i \log_2\left(\frac{|g_i s_i C_u + (\sigma_i^2+\sigma_n^2)\mathbb{1}|}{|(\sigma_i^2+\sigma_n^2)\mathbb{1}|}\right)}{\sum_i \log_2\left(\frac{|s_i C_u + \sigma_i^2\mathbb{1}|}{|\sigma_i^2\mathbb{1}|}\right)}$$

Alternative VIF:

* Vision Model: Wavelet + Gauss. Noise
* Compute $\left\{ \begin{array}{l} I(C,F) \\ I(C,E) \end{array} \right.$ empirically using RBIG

Correlation with Subjective Opinion (TID 2013)

	Pearson	Spearman	Kendall
Conventional VIF	0.78	0.76	0.60
	0.81	0.76	0.59
Alternative RBIG-VIF	0.90	0.87	0.72

Comput. by Benjamin Kheravdar

5 DISCUSION & CONCLUSIONS

5 DISCUSION & CONCLUSIONS

- Info. theory estimates & curse of dimensionality
- Gaussianization $\longrightarrow T(x)$, but still requires $E_x\left[\log_2|\nabla_x G|\right]$
- RBIG $\Rightarrow T(x) \sim$ univariate operations $\Rightarrow$ Alleviates curse dim.
- RBIG $\Rightarrow T, H, D_{KL}, I$ better than Szabo JMLR 2014
- Efficient Coding Hypothesis in Psychophysical Models
    - Deeper represent. are better
    - Psychophysically accurate models are better
    - Space more important than color
    - Transmitted information matches PDF of natural images
- Image Quality: RBIG empirical estimates of I improve VIF

Better information estimates (e.g. using RBIG)

ACADEMY OF TELEVISION
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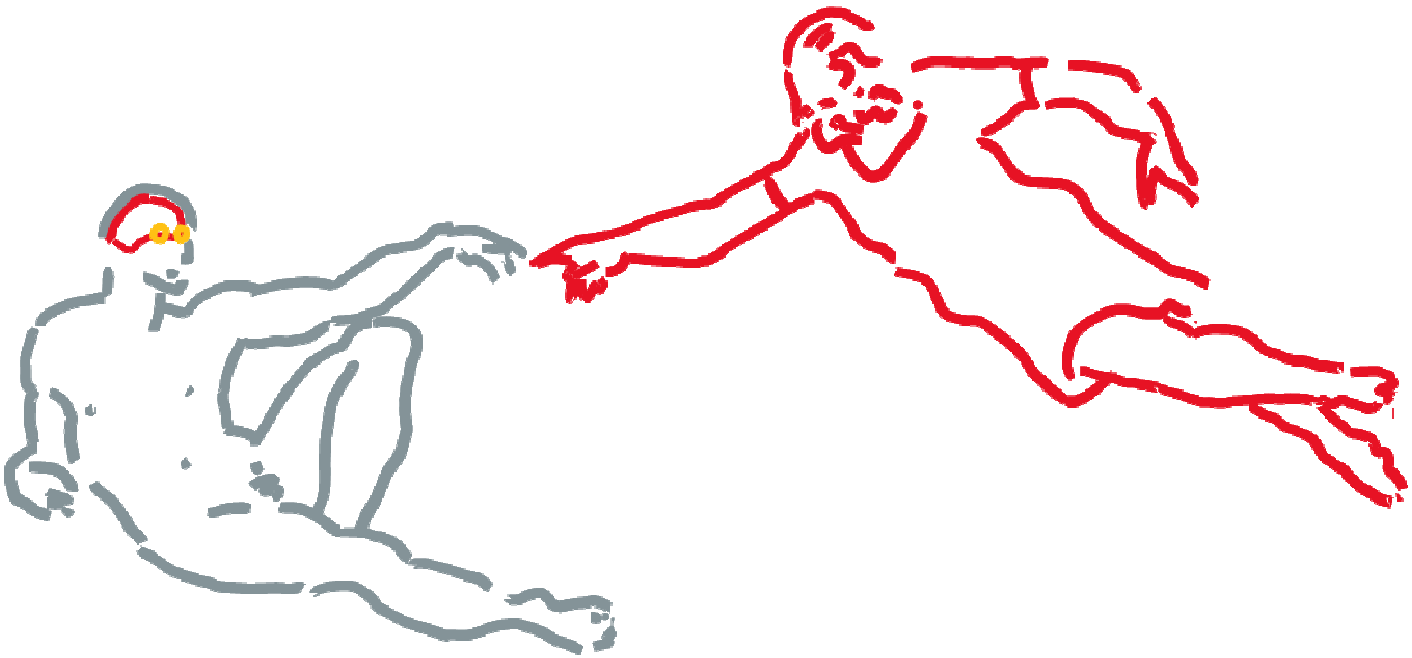
IMAGE QUALITY PROBLEM



EFFICIENT CODING HYPOTHESIS

Better information estimates (e.g. using RBIG)

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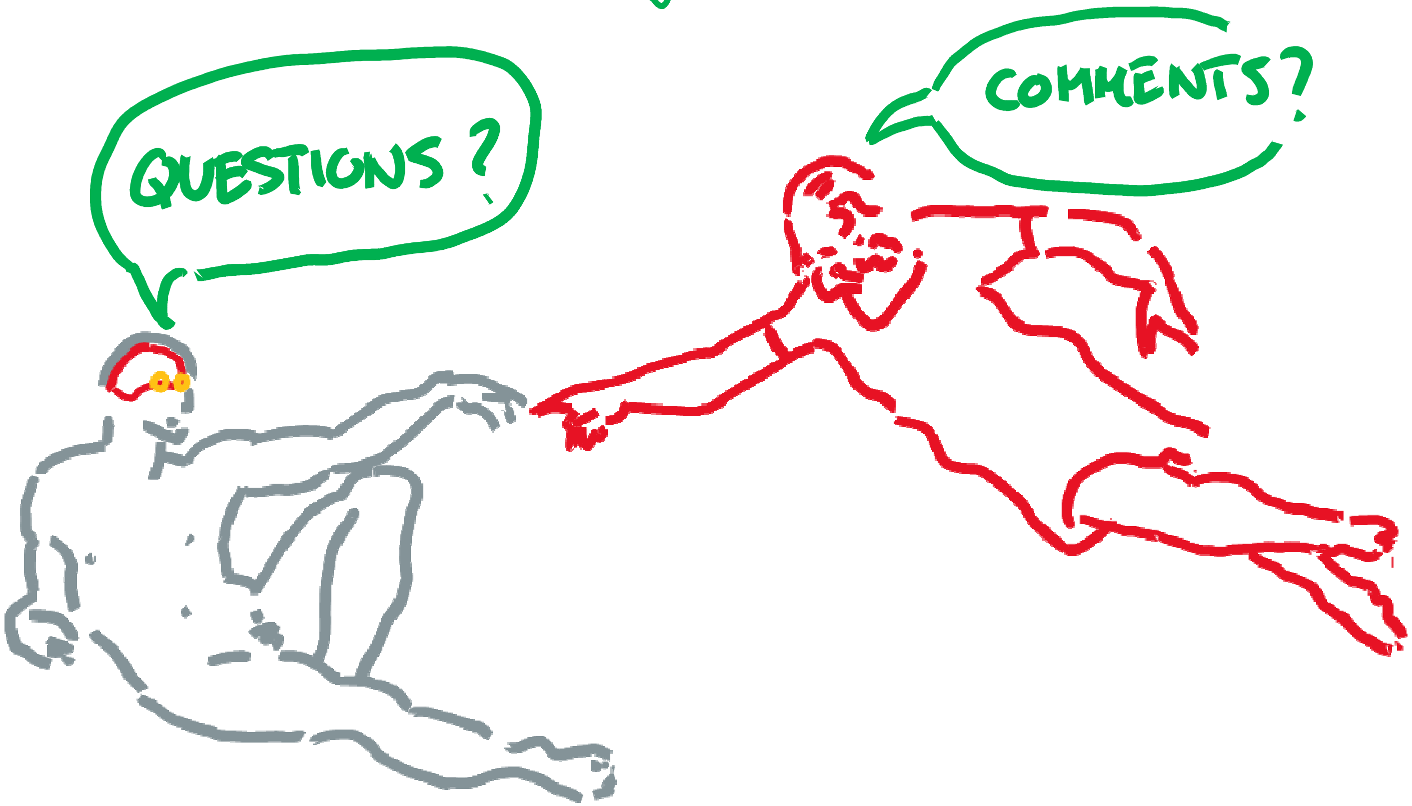
IMAGE QUALITY PROBLEM



EFFICIENT CODING HYPOTHESIS